## Then
- You graphed linear functions.

## Now
- In this chapter, you will:
  - Write and graph linear equations in various forms.
  - Use scatter plots and lines of fit, and write equations of best-fit lines using linear regression.
  - Find inverse linear functions.

## Why?
- **TRAVEL** The number of trips people take changes from year to year. From the yearly data, patterns emerge. Rate of change can be applied to these data to determine a linear model. This can be used to predict the number of trips taken in future years.
Get Ready for the Chapter

Diagnose Readiness | You have two options for checking prerequisite skills.

1 Textbook Option | Take the Quick Check below. Refer to the Quick Review for help.

<table>
<thead>
<tr>
<th>QuickCheck</th>
<th>QuickReview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluate $3a^2 - 2ab + c$ for the values given.</td>
<td>Example 1</td>
</tr>
<tr>
<td>1. $a = 2, b = 1, c = 5$</td>
<td>Evaluate $2(m - n)^2 + 3p$ for $m = 5, n = 2, \text{ and } p = -3$.</td>
</tr>
<tr>
<td>2. $a = -3, b = -2, c = 3$</td>
<td>Original expression</td>
</tr>
<tr>
<td>3. $a = -1, b = 0, c = 11$</td>
<td>Substitute.</td>
</tr>
<tr>
<td>4. $a = 5, b = -3, c = -9$</td>
<td>Subtract.</td>
</tr>
<tr>
<td>5. CAR RENTAL The cost of renting a car is given by $49x + 0.3y$. Let $x$ represent the number of days rented, and let $y$ represent the number of miles driven. Find the cost for a five-day rental over 125 miles.</td>
<td>Evaluate power.</td>
</tr>
</tbody>
</table>

Solve each equation for the given variable.

| 6. $x + y = 5$ for $y$ | Example 2 |
| 7. $2x - 4y = 6$ for $x$ | Solve $5x + 15y = 9$ for $x$. |
| 8. $y - 2 = x + 3$ for $y$ | Original equation |
| 9. $4x - 3y = 12$ for $x$ | Subtract $15y$ from each side. |
| 10. GEOMETRY The formula for the perimeter of a rectangle is $P = 2w + 2l$, where $w$ represents width and $l$ represents length. Solve for $w$. | Simplify. |

Write the ordered pair for each point.

| 11. $A$ | Example 3 |
| 12. $B$ | Write the ordered pair for $A$. |
| 13. $C$ | Step 1 | Begin at point $A$. |
| 14. $D$ | Step 2 | Follow along a vertical line to the $x$-axis. The $x$-coordinate is $-4$. |
| 15. $E$ | Step 3 | Follow along a horizontal line to the $y$-axis. The $y$-coordinate is 2. |
| 16. $F$ | The ordered pair for point $A$ is $(-4, 2)$. |

2 Online Option | Take an online self-check Chapter Readiness Quiz at [connectED.mcgraw-hill.com](http://connectED.mcgraw-hill.com).
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 4. To get ready, identify important terms and organize your resources. You may wish to refer to Chapter 0 to review prerequisite skills.

**New Vocabulary**

<table>
<thead>
<tr>
<th>English</th>
<th>Español</th>
</tr>
</thead>
<tbody>
<tr>
<td>slope-intercept form</td>
<td>forma pendiente-intersección</td>
</tr>
<tr>
<td>linear extrapolation</td>
<td>extrapolación lineal</td>
</tr>
<tr>
<td>point-slope form</td>
<td>forma punto-pendiente</td>
</tr>
<tr>
<td>parallel lines</td>
<td>rectas paralelas</td>
</tr>
<tr>
<td>perpendicular lines</td>
<td>rectas perpendiculares</td>
</tr>
<tr>
<td>scatter plot</td>
<td>gráfica de dispersión</td>
</tr>
<tr>
<td>line of fit</td>
<td>recta de ajuste</td>
</tr>
<tr>
<td>linear interpolation</td>
<td>interpolación lineal</td>
</tr>
<tr>
<td>best-fit line</td>
<td>recta de ajuste óptimo</td>
</tr>
<tr>
<td>linear regression</td>
<td>retroceso lineal</td>
</tr>
<tr>
<td>correlation coefficient</td>
<td>coeficiente de correlación</td>
</tr>
<tr>
<td>median-fit line</td>
<td>línea de mediana-ataque</td>
</tr>
<tr>
<td>inverse relation</td>
<td>relación inversa</td>
</tr>
<tr>
<td>inverse function</td>
<td>función inversa</td>
</tr>
</tbody>
</table>

**Review Vocabulary**

- **coefficient**: the numerical factor of a term
- **function**: a relation in which each element of the domain is paired with exactly one element of the range
- **ratio**: a comparison of two numbers by division

**Foldables Study Organizer**

**Equations of Linear Functions** Make this Foldable to help you organize your Chapter 4 notes about linear functions. Begin with one sheet of 11” by 17” paper.

1. **Fold** each end of the paper in about 2 inches.

2. **Fold** along the width and the length. Unfold. Cut along the fold line from the top to the center.

3. **Fold** the top flaps down. Then fold in half and turn to form a folder. Staple the flaps down to form pockets.

4. **Label** the front with the chapter title.
Set Up the Lab

- Cut a small hole in a top corner of a plastic sandwich bag. Hang the bag from the end of the force sensor.
- Connect the force sensor to your data collection device.

Activity Collect Data

**Step 1** Use the sensor to collect the weight with 0 washers in the bag. Record the data pair in the calculator.

**Step 2** Place one washer in the plastic bag. Wait for the bag to stop swinging, then measure and record the weight.

**Step 3** Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the weight.

Analyze the Results

1. The domain contains values of the independent variable, number of washers. The range contains values of the dependent variable, weight. Use the graphing calculator to create a scatter plot using the ordered pairs (washers, weight).

2. Write a sentence that describes the points on the graph.

3. Describe the position of the point on the graph that represents the trial with no washers in the bag.

4. The rate of change can be found by using the formula for slope.

   \[
   \frac{\text{rise}}{\text{run}} = \frac{\text{change in weight}}{\text{change in number of washers}}
   \]

   Find the rate of change in the weight as more washers are added.

5. Explain how the rate of change is shown on the graph.

Make a Conjecture

The graph shows sample data from a washer experiment. Describe the graph for each situation.

6. a bag that hangs weighs 0.8 N when empty and increases in weight at the rate of the sample

7. a bag that has the same weight when empty as the sample and increases in weight at a faster rate

8. a bag that has the same weight when empty as the sample and increases in weight at a slower rate
1 \textbf{Slope-Intercept Form} An equation of the form $y = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept, is in \textit{slope-intercept form}. The variables $m$ and $b$ are called \textit{parameters} of the equation. Changing either value changes the equation’s graph.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{slope_intercept_form}
\caption{Slope-Intercept Form Diagram}
\end{figure}

\begin{example}
\textbf{Write and Graph an Equation}

Write an equation in slope-intercept form for the line with a slope of $\frac{3}{4}$ and a $y$-intercept of $-2$. Then graph the equation.

\begin{align*}
y &= mx + b \quad \text{Slope-intercept form} \\
y &= \frac{3}{4}x + (-2) \quad \text{Replace } m \text{ with } \frac{3}{4} \text{ and } b \text{ with } -2. \\
y &= \frac{3}{4}x - 2 \quad \text{Simplify.}
\end{align*}

Now graph the equation.

\begin{enumerate}
\item \textbf{Step 1} Plot the $y$-intercept $(0, -2)$.
\item \textbf{Step 2} The slope is $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$. From $(0, -2)$, move up 3 units and right 4 units. Plot the point.
\item \textbf{Step 3} Draw a line through the two points.
\end{enumerate}
\end{example}

\begin{guided}
\textbf{Guided Practice}

Write an equation of a line in slope intercept form with the given slope and $y$-intercept. Then graph the equation.

\begin{enumerate}
\item \textbf{1A.} slope: $-\frac{1}{2}$, $y$-intercept: 3
\item \textbf{1B.} slope: $-3$, $y$-intercept: $-8$
\end{enumerate}
\end{guided}
When an equation is not written in slope-intercept form, it may be easier to rewrite it before graphing.

**Example 2** Graph Linear Equations

Graph $3x + 2y = 6$.

Rewrite the equation in slope-intercept form.

1. $3x + 2y = 6$ \(\text{Original equation}\)
2. $3x + 2y - 3x = 6 - 3x$ \(\text{Subtract 3x from each side.}\)
3. $2y = 6 - 3x$ \(\text{Simplify.}\)
4. $2y = -3x + 6$ \(6 - 3x = 6 + (-3x) \text{ or } -3x + 6\)
5. $\frac{2y}{2} = \frac{-3x + 6}{2}$ \(\text{Divide each side by 2.}\)
6. $y = -\frac{3}{2}x + 3$ \(\text{Slope-intercept form}\)

Now graph the equation. The slope is $-\frac{3}{2}$, and the $y$-intercept is 3.

**Step 1** Plot the $y$-intercept $(0, 3)$.

**Step 2** The slope is $\text{rise} \over \text{run} = -\frac{3}{2}$. From $(0, 3)$, move down 3 units and right 2 units. Plot the point.

**Step 3** Draw a line through the two points.

**Guided Practice**

Graph each equation.

2A. $3x - 4y = 12$  
2B. $-2x + 5y = 10$

Except for the graph of $y = 0$, which lies on the $x$-axis, horizontal lines have a slope of 0. They are graphs of constant functions, which can be written in slope-intercept form as $y = 0x + b$ or $y = b$, where $b$ is any number. Constant functions do not cross the $x$-axis. Their domain is all real numbers, and their range is $b$.

**Example 3** Graph Linear Equations

Graph $y = -3$.

**Step 1** Plot the $y$-intercept $(0, -3)$.

**Step 2** The slope is 0. Draw a line through the points with $y$-coordinate $-3$.

**Guided Practice**

Graph each equation.

3A. $y = 5$  
3B. $2y = 1$

Vertical lines have no slope. So, equations of vertical lines cannot be written in slope-intercept form.
There are times when you will need to write an equation when given a graph. To do this, locate the \( y \)-intercept and use the rise and run to find another point on the graph. Then write the equation in slope-intercept form.

**Test-Taking Tip**

Eliminating Choices

Analyze the graph to determine the slope and the \( y \)-intercept. Then you can save time by eliminating answer choices that do not match the graph.

**Standardized Test Example 4** Write an Equation in Slope-Intercept Form

Which of the following is an equation in slope-intercept form for the line shown?

A \( y = -3x + 1 \)

B \( y = -3x + 3 \)

C \( y = -\frac{1}{3}x + 1 \)

D \( y = -\frac{1}{3}x + 3 \)

**Read the Test Item**

You need to find the slope and \( y \)-intercept of the line to write the equation.

**Solve the Test Item**

**Step 1** The line crosses the \( y \)-axis at \((0, 1)\), so the \( y \)-intercept is 1. The answer is either A or C.

**Step 2** To get from \((0, 1)\) to \((3, 0)\), go down 1 unit and 3 units to the right.

The slope is \(-\frac{1}{3}\).

**Step 3** Write the equation.

\[
y = mx + b
\]

\[
y = -\frac{1}{3}x + 1
\]

**CHECK** The graph also passes through \((-3, 2)\). If the equation is correct, this should be a solution.

\[
y = -\frac{1}{3}x + 1
\]

\[
2 = -\frac{1}{3}(-3) + 1
\]

\[
2 = 1 + 1
\]

\[
2 = 2 \checkmark \quad \text{The answer is C.}
\]

**Guided Practice**

4. Which of the following is an equation in slope-intercept form for the line shown?

F \( y = \frac{1}{4}x - 1 \)

G \( y = \frac{1}{4}x + 4 \)

H \( y = 4x - 1 \)

J \( y = 4x + 4 \)

**Modeling Real-World Data** Real-world data can be modeled by a linear equation if there is a constant rate of change. The rate of change represents the slope. The \( y \)-intercept is the point where the value of the independent variable is 0.
Real-World Example 5 Write and Graph a Linear Equation

**SPORTS** Use the information at the left about high school sports.

a. Write a linear equation to find the number of girls in high school sports after 1997.

<table>
<thead>
<tr>
<th>Words</th>
<th>Number of girls competing equals rate of change times number of years plus amount at start.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td>Let $G$ = number of girls competing. Let $n$ = number of years since 1997.</td>
</tr>
<tr>
<td>Equation</td>
<td>$G = 0.06n + 2.6$</td>
</tr>
</tbody>
</table>

The equation is $G = 0.06n + 2.6$.

b. Graph the equation.

The $y$-intercept is where the data begins. So, the graph passes through (0, 2.6).

The rate of change is the slope, so the slope is 0.06.

c. Estimate the number of girls competing in 2017.

The year 2017 is 20 years after 1997.

$G = 0.06n + 2.6$ Write the equation.

$= 0.06(20) + 2.6$ Replace $n$ with 20.

$= 3.8$ Simplify.

There will be about 3.8 million girls competing in high school sports in 2017.

Guided Practice

5. **FUNDRAISERS** The band boosters are selling sandwiches for $5 each. They bought $1160 in ingredients.

A. Write an equation for the profit $P$ made on $n$ sandwiches.

B. Graph the equation.

C. Find the total profit if 1400 sandwiches are sold.

Check Your Understanding

Example 1 Write an equation of a line in slope-intercept form with the given slope and $y$-intercept. Then graph the equation.

1. slope: 2, $y$-intercept: 4
2. slope: $-5$, $y$-intercept: 3
3. slope: $\frac{3}{4}$, $y$-intercept: $-1$
4. slope: $-\frac{5}{7}$, $y$-intercept: $-\frac{2}{3}$

Examples 2–3 Graph each equation.

5. $-4x + y = 2$
6. $2x + y = -6$
7. $-3x + 7y = 21$
8. $6x - 4y = 16$
9. $y = -1$
10. $15y = 3$
Example 4  Write an equation in slope-intercept form for each graph shown.

11.  

12.  

13.  

14.  

Example 5  15. **FINANCIAL LITERACY**  Rondell is buying a new stereo system for his car using Jack’s Stereo layaway plan.
   a. Write an equation for the total amount $S$ that he has paid after $w$ weeks.
   b. Graph the equation.
   c. Find out how much Rondell will have paid after 8 weeks.

16. **REASONING**  Ana is driving from her home in Miami, Florida, to her grandmother’s house in New York City. On the first day, she will travel 240 miles to Orlando, Florida, to pick up her cousin. Then they will travel 350 miles each day.
   a. Write an equation for the total number of miles $m$ that Ana has traveled after $d$ days.
   b. Graph the equation.
   c. How long will the drive take if the total length of the trip is 1343 miles?

**Practice and Problem Solving**  

Example 1  Write an equation of a line in slope-intercept form with the given slope and $y$-intercept. Then graph the equation.

17. slope: 5, $y$-intercept: 8  
18. slope: 3, $y$-intercept: 10  
19. slope: $-4$, $y$-intercept: 6  
20. slope: $-2$, $y$-intercept: 8  
21. slope: 3, $y$-intercept: $-4$  
22. slope: 4, $y$-intercept: $-6$

Examples 2–3  Graph each equation.

23. $-3x + y = 6$  
24. $-5x + y = 1$  
25. $-2x + y = -4$  
26. $y = 7x - 7$  
27. $5x + 2y = 8$  
28. $4x + 9y = 27$  
29. $y = 7$  
30. $y = -\frac{2}{3}$  
31. $21 = 7y$  
32. $3y - 6 = 2x$
Example 4  Write an equation in slope-intercept form for each graph shown.

Example 5  MANATEES  In 1991, 1267 manatees inhabited Florida’s waters. The manatee
population has increased at a rate of 123 manatees per year.

a. Write an equation for the manatee population, \( P \), \( t \) years since 1991.

b. Graph this equation.

c. In 2006, the manatee was removed from Florida’s endangered species list. What was the manatee population in 2006?

Write an equation of a line in slope-intercept form with the given slope and \( y \)-intercept.

38. slope: \( \frac{1}{2} \), \( y \)-intercept: \(-3\)  
39. slope: \( \frac{2}{3} \), \( y \)-intercept: \(-5\)
40. slope: \( -\frac{5}{6} \), \( y \)-intercept: \(5\)  
41. slope: \( -\frac{3}{7} \), \( y \)-intercept: \(2\)
42. slope: \( 1 \), \( y \)-intercept: \(4\)  
43. slope: \( 0 \), \( y \)-intercept: \(5\)

Graph each equation.

44. \( y = \frac{3}{4}x - 2 \)  
45. \( y = \frac{5}{3}x + 4 \)  
46. \( 3x + 8y = 32 \)
47. \( 5x - 6y = 36 \)  
48. \( -4x + \frac{1}{2}y = -1 \)  
49. \( 3x - \frac{1}{4}y = 2 \)

50. TRAVEL  A rental company charges \$8 per hour for a mountain bike plus
a \$5 fee for a helmet.

a. Write an equation in slope-intercept form for the total rental cost \( C \) for
a helmet and a bicycle for \( t \) hours.

b. Graph the equation.

c. What would the cost be for 2 helmets and 2 bicycles for 8 hours?

51. REASONING  For Illinois residents, the average tuition at Chicago State
University is \$157 per credit hour. Fees cost \$218 per year.

a. Write an equation in slope-intercept form for the tuition \( T \) for \( c \) credit hours.

b. Find the cost for a student who is taking 32 credit hours.
Write an equation of a line in slope-intercept form with the given slope and \( y \)-intercept.

52. slope: \(-1\), \( y \)-intercept: 0  
53. slope: \(0.5\), \( y \)-intercept: 7.5  
54. slope: 0, \( y \)-intercept: 7  
55. slope: \(-1.5\), \( y \)-intercept: \(-0.25\)  
56. Write an equation of a horizontal line that crosses the \( y \)-axis at (0, \(-5\)).
57. Write an equation of a line that passes through the origin and has a slope of 3.

58. **TEMPERATURE** The temperature dropped rapidly overnight. Starting at 80°F, the temperature dropped 3° per minute.
   a. Draw a graph that represents this drop from 0 to 8 minutes.
   b. Write an equation that describes this situation. Describe the meaning of each variable as well as the slope and \( y \)-intercept.

59. **FITNESS** Refer to the information at the right.
   a. Write an equation that represents the cost \( C \) of a membership for \( m \) months.
   b. What does the slope represent?
   c. What does the \( C \)-intercept represent?
   d. What is the cost of a two-year membership?

60. **MAGAZINES** A teen magazine began with a circulation of 500,000 in its first year. Since then, the circulation has increased an average of 33,388 per year.
   a. Write an equation that represents the circulation \( c \) after \( t \) years.
   b. What does the slope represent?
   c. What does the \( c \)-intercept represent?
   d. If the magazine began in 1944, and this trend continues, in what year will the circulation reach 3,000,000?

61. **SMART PHONES** A telecommunications company sold 3305 smart phones in the first year of production. Suppose, on average, they expect to sell 25 phones per day.
   a. Write an equation for the number of smart phones \( P \) sold \( t \) years after the first year of production, assuming 365 days per year.
   b. If sales continue at this rate, how many years will it take for the company to sell 100,000 phones?

**H.O.T. Problems** Use Higher-Order Thinking Skills

62. **OPEN ENDED** Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents.

63. **REASONING** Determine whether the equation of a vertical line can be written in slope-intercept form. Explain your reasoning.

64. **CHALLENGE** Summarize the characteristics that the graphs \( y = 2x + 3 \), \( y = 4x + 3 \), \( y = -x + 3 \), and \( y = -10x + 3 \) have in common.

65. **CSS REGULARITY** If given an equation in standard form, explain how to determine the rate of change.

66. **WRITING IN MATH** Explain how you would use a given \( y \)-intercept and the slope to predict a \( y \)-value for a given \( x \)-value without graphing.
67. A music store has $x$ CDs in stock. If 350 are sold and 3$y$ are added to stock, which expression represents the number of CDs in stock?

A $350 + 3y - x$  
B $x - 350 + 3y$  
C $x + 350 + 3y$  
D $3y - 350 - x$

68. **PROBABILITY** The table shows the result of a survey of favorite activities. What is the probability that a student’s favorite activity is sports or drama club?

<table>
<thead>
<tr>
<th>Extracurricular Activity</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>art club</td>
<td>24</td>
</tr>
<tr>
<td>band</td>
<td>134</td>
</tr>
<tr>
<td>choir</td>
<td>37</td>
</tr>
<tr>
<td>drama club</td>
<td>46</td>
</tr>
<tr>
<td>mock trial</td>
<td>19</td>
</tr>
<tr>
<td>school paper</td>
<td>26</td>
</tr>
<tr>
<td>sports</td>
<td>314</td>
</tr>
</tbody>
</table>

- F $\frac{3}{8}$  
- G $\frac{4}{9}$  
- H $\frac{3}{5}$  
- J $\frac{2}{3}$

69. A recipe for fruit punch calls for 2 ounces of orange juice for every 8 ounces of lemonade. If Jennifer uses 64 ounces of lemonade, which proportion can she use to find $x$, the number of ounces of orange juice needed?

A $\frac{2}{x} = \frac{64}{6}$  
B $\frac{8}{x} = \frac{64}{2}$  
C $\frac{2}{8} = \frac{x}{64}$  
D $\frac{6}{2} = \frac{x}{64}$

70. **EXTENDED RESPONSE** The table shows the results of a canned food drive. 1225 cans were collected, and the 12th-grade class collected 55 more cans than the 10th-grade class. How many cans each did the 10th- and 12th-grade classes collect? Show your work.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Cans</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>340</td>
</tr>
<tr>
<td>10</td>
<td>$x$</td>
</tr>
<tr>
<td>11</td>
<td>280</td>
</tr>
<tr>
<td>12</td>
<td>$y$</td>
</tr>
</tbody>
</table>

71. For each arithmetic sequence, determine the related function. Then determine if the function is proportional or nonproportional. (Lesson 3-6)

71. 3, 7, 11, …  
72. 8, 6, 4, …  
73. 0, 3, 6, …  
74. 1, 2, 3, …

75. **GAME SHOWS** Contestants on a game show win money by answering 10 questions. (Lesson 3-5)

a. Find the value of the 10th question.

b. If all questions are answered correctly, how much are the winnings?

Suppose $y$ varies directly as $x$. Write a direct variation equation that relates $x$ and $y$. Then solve. (Lesson 3-4)

76. If $y = 10$ when $x = 5$, find $y$ when $x = 6$.

77. If $y = -16$ when $x = 4$, find $x$ when $y = 20$.

78. If $y = 6$ when $x = 18$, find $y$ when $x = -12$.

79. If $y = 12$ when $x = 15$, find $x$ when $y = -6$.

76. If $y = 10$ when $x = 5$, find $y$ when $x = 6$.

77. If $y = -16$ when $x = 4$, find $x$ when $y = 20$.

78. If $y = 6$ when $x = 18$, find $y$ when $x = -12$.

79. If $y = 12$ when $x = 15$, find $x$ when $y = -6$.

76. If $y = 10$ when $x = 5$, find $y$ when $x = 6$.

77. If $y = -16$ when $x = 4$, find $x$ when $y = 20$.

78. If $y = 6$ when $x = 18$, find $y$ when $x = -12$.

79. If $y = 12$ when $x = 15$, find $x$ when $y = -6$.

80. (2, 3), (9, 7)  
81. (−3, 6), (2, 4)  
82. (2, 6), (−1, 3)  
83. (−3, 3), (1, 3)
Graphing Technology Lab

The Family of Linear Graphs

A family of people is related by birth, marriage, or adoption. Often people in families share characteristics. The graphs in a family share at least one characteristic.

Graphs in the linear family are all lines, with the simplest graph in the family being that of the parent function \( y = x \). This parent function is also known as the identity function. Its graph contains all points with coordinates \((a, a)\). Its domain and range are all real numbers.

You can use a graphing calculator to investigate how changing the parameters \( m \) and \( b \) in \( y = mx + b \) affects the graphs in the family of linear functions.

**Activity 1 Changing \( b \) in \( y = mx + b \)**

Graph \( y = x \), \( y = x + 4 \), and \( y = x - 2 \) in the standard viewing window.

Enter the equations in the \( Y= \) list as \( Y_1 \), \( Y_2 \), and \( Y_3 \). Then graph the equations.

**KEYSTROKES:**

\[
\begin{align*}
Y_1 &= X, T, \theta, n \quad \text{ENTER} \\
Y_2 &= 4 \quad \text{ENTER} \\
\text{ZOOM} &= 6
\end{align*}
\]

1A. How do the slopes of the graphs compare?

1B. Compare the graph of \( y = x + 4 \) and the graph of \( y = x \). How would you obtain the graph of \( y = x + 4 \) from the graph of \( y = x \)?

1C. How would you obtain the graph of \( y = x - 2 \) from the graph of \( y = x \)?

Changing the \( y \)-intercept, \( b \), translates, or moves, a linear function up or down the \( y \)-axis. Changing \( m \) in \( y = mx + b \) affects the graphs in a different way. First, investigate positive values of \( m \).

**Activity 2 Changing \( m \) in \( y = mx + b \), Positive Values**

Graph \( y = x \), \( y = 2x \), and \( y = \frac{1}{3}x \) in the standard viewing window.

Enter the equations in the \( Y= \) list and graph.

2A. How do the \( y \)-intercepts of the graphs compare?

2B. Compare the graph of \( y = 2x \) and the graph of \( y = x \).

2C. Which is steeper, the graph of \( y = \frac{1}{3}x \) or the graph of \( y = x \)?

Does changing \( m \) to a negative value affect the graph differently than changing it to a positive value?
Graph $y = x, y = -x, y = -3x,$ and $y = -\frac{1}{2}x$ in the standard viewing window.

Enter the equations in the $Y =$ list and graph.

3A. How are the graphs with negative values of $m$ different than graphs with a positive $m$?
3B. Compare the graphs of $y = -x, y = -3x,$ and $y = -\frac{1}{2}x$. Which is steepest?

Analyze the Results

**SENSE-MAKING AND PERSEVERANCE** Graph each set of equations on the same screen.

Describe the similarities or differences.

1. $y = 2x$
   $y = 2x + 3$
   $y = 2x - 7$

2. $y = x + 1$
   $y = 2x + 1$
   $y = \frac{1}{4}x + 1$

3. $y = x + 4$
   $y = 2x + 4$
   $y = \frac{3}{4}x + 4$

4. $y = 0.5x + 2$
   $y = 0.5x - 5$
   $y = 0.5x + 4$

5. $y = -2x - 2$
   $y = -4.2x - 2$
   $y = -\frac{1}{3}x - 2$

6. $y = 3x$
   $y = 3x + 6$
   $y = 3x - 7$

7. Families of graphs have common characteristics. What do the graphs of all equations of the form $y = mx + b$ have in common?

8. How does the value of $b$ affect the graph of $y = mx + b$?

9. What is the result of changing the value of $m$ on the graph of $y = mx + b$ if $m$ is positive?

10. How can you determine which graph is steepest by examining the following equations?
    $y = 3x, y = -4x - 7, y = \frac{1}{2}x + 4$

11. Explain how knowing about the effects of $m$ and $b$ can help you sketch the graph of an equation.

12. The equation $y = k$ can also be a parent graph. Graph $y = 5, y = 2,$ and $y = -4$ on the same screen. Describe the similarities or differences among the graphs.

**Extension**

Nonlinear functions can also be defined in terms of a family of graphs. Graph each set of equations on the same screen. Describe the similarities or differences.

13. $y = x^2$
    $y = -3x^2$
    $y = (-3x)^2$

14. $y = x^2$
    $y = x^2 + 3$
    $y = (x - 2)^2$

15. $y = x^2$
    $y = 2x^2 + 4$
    $y = (3x)^2 - 5$

16. Describe the similarities and differences in the classes of functions $f(x) = x^2 + c$ and $f(x) = (x + c)^2$, where $c$ is any real number.
Lesson 4-2

Writing Equations in Slope-Intercept Form

1 Write an Equation Given the Slope and a Point

The next example shows how to write an equation of a line if you are given a slope and a point other than the y-intercept.

**Example 1** Write an Equation Given the Slope and a Point

Write an equation of the line that passes through (2, 1) with a slope of 3.

You are given the slope but not the y-intercept.

**Step 1** Find the y-intercept.

\[ y = mx + b \] Slope-intercept form

\[ 1 = 3(2) + b \]

Replace \( m \) with 3, \( y \) with 1, and \( x \) with 2.

\[ 1 = 6 + b \]

Simplify.

\[ 1 - 6 = 6 + b - 6 \]

Subtract 6 from each side.

\[ -5 = b \]

Simplify.

**Step 2** Write the equation in slope-intercept form.

\[ y = mx + b \] Slope-intercept form

\[ y = 3x - 5 \]

Replace \( m \) with 3 and \( b \) with -5.

Therefore, the equation of the line is \( y = 3x - 5 \).

**Guided Practice**

Write an equation of a line that passes through the given point and has the given slope.

1A. \((-2, 5), \text{ slope } 3\)  
1B. \((4, -7), \text{ slope } -1\)

2 Write an Equation Given Two Points

If you are given two points through which a line passes, you can use them to find the slope first. Then follow the steps in Example 1 to write the equation.
Example 2 Write an Equation Given Two Points

Write an equation of the line that passes through each pair of points.

a. (3, 1) and (2, 4)

Step 1 Find the slope of the line containing the given points.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \frac{4 - 1}{2 - 3} \]

\[ m = \frac{3}{-1} \text{ or } -3 \]

Simplify.

Step 2 Use either point to find the y-intercept.

\[ y = mx + b \]

\[ 4 = (-3)(2) + b \]

Replace \( m \) with \(-3\), \( x \) with 2, and \( y \) with 4.

\[ 4 = -6 + b \]

Simplify.

\[ 4 - (-6) = -6 + b - (-6) \]

Subtract \(-6\) from each side.

\[ 10 = b \]

Simplify.

Step 3 Write the equation in slope-intercept form.

\[ y = mx + b \]

\[ y = -3x + 10 \]

Replace \( m \) with \(-3\) and \( b \) with 10.

Therefore, the equation is \( y = -3x + 10 \).

b. (−4, −2) and (−5, −6)

Step 1 Find the slope of the line containing the given points.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \frac{-6 - (-2)}{-5 - (-4)} \]

\[ m = \frac{-4}{1} \text{ or } 4 \]

Simplify.

Step 2 Use either point to find the y-intercept.

\[ y = mx + b \]

\[ -2 = 4(-4) + b \]

Replace \( m \) with 4, \( x \) with \(-4\), and \( y \) with \(-2\).

\[ -2 = -16 + b \]

Simplify.

\[ -2 - (-16) = -16 + b - (-16) \]

Subtract \(-16\) from each side.

\[ 14 = b \]

Simplify.

Step 3 Write the equation in slope-intercept form.

\[ y = mx + b \]

\[ y = 4x + 14 \]

Replace \( m \) with 4 and \( b \) with 14.

Therefore, the equation is \( y = 4x + 14 \).

Guided Practice

Write an equation of the line that passes through each pair of points.

2A. (−1, 12), (4, −8)  2B. (5, −8), (−7, 0)
In mathematics, a constraint is a condition that a solution must satisfy. Equations can be viewed as constraints in a problem situation. The solutions of the equation meet the constraints of the problem.

**Real-World Example 3  Use Slope-Intercept Form**

**FLIGHTS** The table shows the number of domestic flights in the U.S. from 2004 to 2008. Write an equation that could be used to predict the number of flights if it continues to decrease at the same rate.

<table>
<thead>
<tr>
<th>Year</th>
<th>Flights (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>9.97</td>
</tr>
<tr>
<td>2005</td>
<td>10.04</td>
</tr>
<tr>
<td>2006</td>
<td>9.71</td>
</tr>
<tr>
<td>2007</td>
<td>9.84</td>
</tr>
<tr>
<td>2008</td>
<td>9.37</td>
</tr>
</tbody>
</table>

**Understand** You know the number of flights for 2004–2008.

**Plan** Let \( x \) represent the number of years since 2000, and let \( y \) represent the number of flights. Write an equation of the line that passes through (4, 9.97) and (8, 9.37).

**Solve** Find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9.37 - 9.97}{8 - 4} = \frac{-0.6}{4} = -0.15
\]

Use (8, 9.37) to find the \( y \)-intercept of the line.

\[
y = mx + b
\]

\[
9.37 = -0.15(8) + b
\]

\[
9.37 = -1.2 + b
\]

\[
10.57 = b
\]

Write the equation using \( m = -0.15 \) and \( b = 10.57 \).

\[
y = -0.15x + 10.57
\]

**Check** Check your result by using the coordinates of the other point.

\[
y = -0.15x + 10.57
\]

\[
9.97 = -0.15(4) + 10.57
\]

\[
9.97 = 9.97
\]

**Guided Practice**

3. **FINANCIAL LITERACY** In addition to his weekly salary, Ethan is paid $16 per delivery. Last week, he made 5 deliveries, and his total pay was $215. Write a linear equation to find Ethan’s total weekly pay \( T \) if he makes \( d \) deliveries.

You can use a linear equation to make predictions about values that are beyond the range of the data. This process is called **linear extrapolation**.

**Real-World Example 4  Predict from Slope-Intercept Form**

**FLIGHTS** Estimate the number of domestic flights in 2020.

\[
y = -0.15x + 10.57
\]

\[
y = -0.15(20) + 10.57 \text{ or } 7.57 \text{ million}
\]

**Guided Practice**

4. **MONEY** Use the equation in Guided Practice 3 to predict how much money Ethan will earn in a week if he makes 8 deliveries.
Check Your Understanding

Example 1  Write an equation of the line that passes through the given point and has the given slope.

1. (3, -3), slope 3  
2. (2, 4), slope 2  
3. (1, 5), slope -1  
4. (-4, 6), slope -2

Example 2  Write an equation of the line that passes through each pair of points.

5. (4, -3), (2, 3)  
6. (-7, -3), (-3, 5)  
7. (-1, 3), (0, 8)  
8. (-2, 6), (0, 0)

Examples 3, 4  9. **WHITESTR WATER RAFTING** Ten people from a local youth group went to Black Hills Whitewater Rafting Tour Company for a one-day rafting trip. The group paid $425.

a. Write an equation in slope-intercept form to find the total cost $C$ for $p$ people.

b. How much would it cost for 15 people?

Practice and Problem Solving

Example 1  Write an equation of the line that passes through the given point and has the given slope.

10. (3, 1), slope 2  
11. (-1, 4), slope -1  
12. (1, 0), slope 1  
13. (7, 1), slope 8  
14. (2, 5), slope -2  
15. (2, 6), slope 2

Example 2  Write an equation of the line that passes through each pair of points.

16. (9, -2), (4, 3)  
17. (-2, 5), (5, -2)  
18. (-5, 3), (0, -7)  
19. (3, 5), (2, -2)  
20. (-1, -3), (-2, 3)  
21. (-2, -4), (2, 4)

Examples 3, 4  22. **CSS MODELING** Greg is driving a remote control car at a constant speed. He starts the timer when the car is 5 feet away. After 2 seconds the car is 35 feet away.

a. Write a linear equation to find the distance $d$ of the car from Greg.

b. Estimate the distance the car has traveled after 10 seconds.

23. ZOOS  Refer to the beginning of the lesson.

a. Write a linear equation to find the attendance (in millions) $y$ after $x$ years. Let $x$ be the number of years since 2000.

b. Estimate the zoo’s attendance in 2020.

24. BOOKS  In 1904, a dictionary cost 30¢. Since then the cost of a dictionary has risen an average of 6¢ per year.

a. Write a linear equation to find the cost $C$ of a dictionary $y$ years after 1904.

b. If this trend continues, what will the cost of a dictionary be in 2020?

Write an equation of the line that passes through the given point and has the given slope.

25. (4, 2), slope $\frac{1}{2}$  
26. (3, -2), slope $\frac{1}{3}$  
27. (6, 4), slope $-\frac{3}{4}$  
28. (2, -3), slope $\frac{2}{3}$  
29. (2, -2), slope $\frac{2}{7}$  
30. (-4, -2), slope $-\frac{3}{5}$
31. **DOGS**  In 2001, there were about 56.1 thousand golden retrievers registered in the United States. In 2002, the number was 62.5 thousand.
   a. Write a linear equation to find the number of thousands of golden retrievers \( G \) that will be registered in year \( t \), where \( t = 0 \) is the year 2000.
   b. Graph the equation.
   c. Estimate the number of golden retrievers that will be registered in 2017.

32. **GYM MEMBERSHIPS**  A local recreation center offers a yearly membership for $265. The center offers aerobics classes for an additional $5 per class.
   a. Write an equation that represents the total cost of the membership.
   b. Carly spent $500 one year. How many aerobics classes did she take?

33. **SUBSCRIPTION**  A magazine offers an online subscription that allows you to view up to 25 archived articles free. To view 30 archived articles, you pay $49.15. To view 33 archived articles, you pay $57.40.
   a. What is the cost of each archived article for which you pay a fee?
   b. What is the cost of the magazine subscription?

Write an equation of the line that passes through the given points.

34. \((5, -2), (7, 1)\)
35. \((5, -3), (2, 5)\)
36. \((\frac{5}{4}, 1), (-\frac{1}{4}, \frac{3}{4})\)
37. \((\frac{5}{12}, -1), (-\frac{3}{4}, \frac{1}{6})\)

Determine whether the given point is on the line. Explain why or why not.

38. \((3, -1); y = \frac{1}{3}x + 5\)
39. \((6, -2); y = \frac{1}{2}x - 5\)

For Exercises 40–42, determine which equation best represents each situation. Explain the meaning of each variable.

- **A** \( y = -\frac{1}{3}x + 72 \)
- **B** \( y = 2x + 225 \)
- **C** \( y = 8x + 4 \)

40. **CONCERTS**  Tickets to a concert cost $8 each plus a processing fee of $4 per order.
41. **FUNDRAISING**  The freshman class has $225. They sell raffle tickets at $2 each to raise money for a field trip.
42. **POOLS**  The current water level of a swimming pool in Tucson, Arizona, is 6 feet. The rate of evaporation is \( \frac{1}{3} \) inch per day.
43. **SENSE-MAKING**  A manufacturer implemented a program to reduce waste. In 1998 they sent 946 tons of waste to landfills. Each year after that, they reduced their waste by an average 28.4 tons.
   a. How many tons were sent to the landfill in 2010?
   b. In what year will it become impossible for this trend to continue? Explain.
44. **COMBINING FUNCTIONS**  The parents of a college student open an account for her with a deposit of $5000, and they set up automatic deposits of $100 to the account every week.
   a. Write a function \( d(t) \) to express the amount of money in the account \( t \) weeks after the initial deposit.
   b. The student plans on spending $600 the first week and $250 in each of the following weeks for room and board and other expenses. Write a function \( w(t) \) to express the amount of money taken out of the account each week.
   c. Find \( B(t) = d(t) - w(t) \). What does this new function represent?
   d. Will the student run out of money? If so, when?
**CONCERT TICKETS** Jackson is ordering tickets for a concert online. There is a processing fee for each order, and the tickets are $52 each. Jackson ordered 5 tickets and the cost was $275.

a. Determine the processing fee. Write a linear equation to represent the total cost C for t tickets.

b. Make a table of values for at least three other numbers of tickets.

c. Graph this equation. Predict the cost of 8 tickets.

**MUSIC** A music store is offering a Frequent Buyers Club membership. The membership costs $22 per year, and then a member can buy CDs at a reduced price. If a member buys 17 CDs in one year, the cost is $111.25.

a. Determine the cost of each CD for a member.

b. Write a linear equation to represent the total cost y of a one year membership, if x CDs are purchased.

c. Graph this equation.

---

**H.O.T. Problems** Use Higher-Order Thinking Skills

**47. ERROR ANALYSIS** Tess and Jacinta are writing an equation of the line through (3, -2) and (6, 4). Is either of them correct? Explain your reasoning.

**Tess**
\[ m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3} = 2 \]
\[ y = mx + b \]
\[ 6 = 2(3) + b \]
\[ 6 = 8 + b \]
\[ -2 = b \]
\[ y = 2x - 2 \]

**Jacinta**
\[ m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3} = 2 \]
\[ y = mx + b \]
\[ -2 = 2(3) + b \]
\[ -2 = 6 + b \]
\[ -8 = b \]
\[ y = 2x - 8 \]

**48. CHALLENGE** Consider three points, (3, 7), (-6, 1) and (9, p), on the same line. Find the value of p and explain your steps.

**49. REASONING** Consider the standard form of a linear equation, \( Ax + By = C \).

a. Rewrite the equation in slope-intercept form.

b. What is the slope?

c. What is the \( y \)-intercept?

d. Is this true for all real values of \( A \), \( B \), and \( C \)?

**50. OPEN ENDED** Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship and describe what the slope and \( y \)-intercept mean.

**51. WRITING IN MATH** Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making any predictions.

**52. CSST ARGUMENTS** What information is needed to write the equation of a line? Explain.
53. Which equation best represents the graph?  
   - A \( y = 2x \)  
   - B \( y = -2x \)  
   - C \( y = \frac{1}{2}x \)  
   - D \( y = -\frac{1}{2}x \)  

54. Roberto receives an employee discount of 12%. If he buys a $355 item at the store, what is his discount to the nearest dollar?  
   - F $3  
   - H $30  
   - G $4  
   - J $43  

55. GEOMETRY The midpoints of the sides of the large square are joined to form a smaller square. What is the area of the smaller square?  
   - A 64 cm²  
   - B 128 cm²  
   - C 248 cm²  
   - D 256 cm²  

56. SHORT RESPONSE If \( \frac{5(x + 4)}{2} + 7 = 37 \), what is the value of \( 3x - 9 \)?  

---

Spiral Review

Graph each equation. (Lesson 4-1)

57. \( y = 3x + 2 \)  
58. \( y = -4x + 2 \)  
59. \( 3y = 2x + 6 \)  
60. \( y = \frac{1}{2}x + 6 \)  
61. \( 3x + y = -1 \)  
62. \( 2x + 3y = 6 \)  

Write an equation in function notation for each relation. (Lesson 3-6)

63.  
64.  

65. METEOROLOGY The distance \( d \) in miles that the sound of thunder travels in \( t \) seconds is given by the equation \( d = 0.21t \). (Lesson 3-4)
   a. Graph the equation.  
   b. Use the graph to estimate how long it will take you to hear thunder from a storm 3 miles away.

Solve each equation. Check your solution. (Lesson 2-3)

66. \( -5t - 2.2 = -2.9 \)  
67. \( -5.5a - 43.9 = 77.1 \)  
68. \( 4.2r + 7.14 = 12.6 \)  
69. \( -14 - \frac{n}{9} = 9 \)  
70. \( \frac{-8b - (-9)}{-10} = 17 \)  
71. \( 9.5x + 11 - 7.5x = 14 \)  

Skills Review

Find the value of \( r \) so the line through each pair of points has the given slope.

72. \((6, -2), (r, -6), m = 4\)  
73. \((8, 10), (r, 4), m = 6\)  
74. \((7, -10), (r, 4), m = -3\)  
75. \((6, 2), (9, r), m = -1\)  
76. \((9, r), (6, 3), m = -\frac{1}{3}\)  
77. \((5, r), (2, -3), m = \frac{4}{3}\)
You wrote linear equations given either one point and the slope or two points.

1. Write equations of lines in point-slope form.
2. Write linear equations in different forms.

Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by

\[ y = 3000x + 200, \]

where \( x \) is the number of months and \( y \) is the number of animals.

**Key Concept**

**Point-Slope Form**

An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.

The linear equation

\[ y - y_1 = m(x - x_1) \]

is written in point-slope form, where \((x_1, y_1)\) is a given point on a nonvertical line and \( m \) is the slope of the line.

**Example 1** Write and Graph an Equation in Point-Slope Form

Write an equation in point-slope form for the line that passes through \((3, -2)\) with a slope of \(\frac{1}{4}\). Then graph the equation.

\[ y - y_1 = m(x - x_1) \quad \text{Point-slope form} \]

\[ y - (-2) = \frac{1}{4}(x - 3) \quad (x_1, y_1) = (3, -2), m = \frac{1}{4} \]

\[ y + 2 = \frac{1}{4}(x - 3) \quad \text{Simplify.} \]

Plot the point at \((3, -2)\) and use the slope to find another point on the line. Draw a line through the two points.

**Guided Practice**

1. Write an equation in point-slope form for the line that passes through \((-2, 1)\) with a slope of \(-6\). Then graph the equation.
**Forms of Linear Equations** If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

### Concept Summary: Writing Equations

<table>
<thead>
<tr>
<th><strong>Given the Slope and One Point</strong></th>
<th><strong>Given Two Points</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong> Substitute the value of $m$ and let the $x$ and $y$ coordinates be $(x_1, y_1)$. Or, substitute the value of $m$, $x$, and $y$ into the slope-intercept form and solve for $b$.</td>
<td><strong>Step 1</strong> Find the slope.</td>
</tr>
<tr>
<td><strong>Step 2</strong> Rewrite the equation in the needed form.</td>
<td><strong>Step 2</strong> Choose one of the two points to use.</td>
</tr>
<tr>
<td><strong>Step 3</strong> Follow the steps for writing an equation given the slope and one point.</td>
<td><strong>Step 3</strong> Follow the steps for writing an equation given the slope and one point.</td>
</tr>
</tbody>
</table>

---

**Example 2** Standard Form

Write $y - 1 = -\frac{2}{3}(x - 5)$ in standard form.

$y - 1 = -\frac{2}{3}(x - 5)$  
$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5)$  
$3(y - 1) = -2(x - 5)$  
$3y - 3 = -2x + 10$  
$3y = -2x + 13$  
$2x + 3y = 13$

**Guided Practice**

2. Write $y - 1 = 7(x + 5)$ in standard form.

---

To find the $y$-intercept of an equation, rewrite the equation in slope-intercept form.

**Example 3** Slope-Intercept Form

Write $y + 3 = \frac{3}{2}(x + 1)$ in slope-intercept form.

$y + 3 = \frac{3}{2}(x + 1)$  
$y + 3 = \frac{3}{2}x + \frac{3}{2}$  
$y = \frac{3}{2}x - \frac{3}{2}$

**Guided Practice**

3. Write $y + 6 = -3(x - 4)$ in slope-intercept form.
Example 4  Point-Slope Form and Standard Form

**GEOMETRY**  The figure shows square RSTU.

a. Write an equation in point-slope form for the line containing side TU.

**Step 1**  Find the slope of TU.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Slope Formula

\[ = \frac{5 - 2}{7 - 4} \text{ or } 1 \]

\((x_1, y_1) = (4, 2) \) and \((x_2, y_2) = (7, 5)\)

**Step 2**  You can select either point for \((x_1, y_1)\) in the point-slope form.

\[ y - y_1 = m(x - x_1) \]

Point-slope form

\[ y - 2 = 1(x - 4) \]

\((x_1, y_1) = (4, 2)\)

\[ y - 5 = 1(x - 7) \]

\((x_1, y_1) = (7, 5)\)

b. Write an equation in standard form for the same line.

\[ y - 2 = 1(x - 4) \]

Original equation

\[ y - 5 = 1(x - 7) \]

Distributive Property

\[ y = 1x - 2 \]

Add to each side.

\[ -1x + y = -2 \]

Subtract 1x from each side.

\[ x - y = 2 \]

Multiply each side by -1.

Guided Practice

4A. Write an equation in point-slope form of the line containing side ST.

4B. Write an equation in standard form of the line containing ST.
Example 4 10. **GEOMETRY** Use right triangle $FGH$.
   
a. Write an equation in point-slope form for the line containing $\overline{GH}$.
   
b. Write the standard form of the line containing $\overline{GH}$.

**Practice and Problem Solving**

Example 1 Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

11. $(5, 3), m = 7$
12. $(2, -1), m = -3$
13. $(-6, -3), m = -1$
14. $(-7, 6), m = 0$
15. $(-2, 11), m = \frac{4}{3}$
16. $(-6, -8), m = -\frac{5}{8}$
17. $(-2, -9), m = -\frac{7}{5}$
18. $(-6, 0)$, horizontal line

Example 2 Write each equation in standard form.

19. $y - 10 = 2(x - 8)$
20. $y - 6 = -3(x + 2)$
21. $y - 9 = -6(x + 9)$
22. $y + 4 = \frac{2}{3}(x + 7)$
23. $y + 7 = \frac{9}{10}(x + 3)$
24. $y + 7 = -\frac{3}{2}(x + 1)$
25. $2y + 3 = -\frac{1}{3}(x - 2)$
26. $4y - 5x = 3(4x - 2y + 1)$

Example 3 Write each equation in slope-intercept form.

27. $y - 6 = -2(x - 7)$
28. $y - 11 = 3(x + 4)$
29. $y + 5 = -6(x + 7)$
30. $y - 1 = \frac{4}{5}(x + 5)$
31. $y + 2 = \frac{1}{6}(x - 4)$
32. $y + 6 = -\frac{3}{4}(x + 8)$
33. $y + 3 = -\frac{1}{3}(2x + 6)$
34. $y + 4 = 3(3x + 3)$

Example 4 **MOVIE RENTALS** The number of copies of a movie rented at a video kiosk decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?

35 **REASONING** A company offers premium cable for $39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is $264.70.

a. Write an equation in point-slope form to find the total price $y$ for any number of months $x$. (Hint: The point (6, 264.70) is a solution to the equation.)

b. Write the equation in slope-intercept form.

b. What is the setup fee?

Write an equation for the line described in standard form.

36. through $(-1, 7)$ and $(8, -2)$
37. through $(-4, 3)$ with $y$-intercept 0
38. with $x$-intercept 4 and $y$-intercept 5

236 | Lesson 4-3 | Writing Equations in Point-Slope Form
Write an equation in point-slope form for each line.

40. \((1, 3)\) \(m = 4\)

41. \((-4, -1)\) \(m = \frac{3}{2}\)

42. \((-3, 7)\) \(m = -\frac{4}{5}\)

Write each equation in slope-intercept form.

43. \(y + \frac{3}{5} = x - \frac{2}{5}\)

44. \(y - \frac{7}{2} = \frac{1}{2}(x - 4)\)

45. \(y + \frac{1}{3} = \frac{5}{6}(x + \frac{2}{5})\)

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through \((-2, 8)\) with slope \(\frac{8}{5}\).

47. Line \(\ell\) passes through \((-9, 4)\) with slope \(\frac{4}{7}\). Write an equation in point-slope form, slope-intercept form, and standard form for line \(\ell\).

48. **WEATHER** The barometric pressure is 598 millimeters of mercury (mmHg) at an altitude of 1.8 kilometers and 577 millimeters of mercury at 2.1 kilometers.
   a. Write a formula for the barometric pressure as a function of the altitude.
   b. What is the altitude if the pressure is 657 millimeters of mercury?

**H.O.T. Problems** Use Higher-Order Thinking Skills

49. WHICH ONE DOESN’T BELONG? Identify the equation that does not belong. Explain your reasoning.
   
   
   \[y - 5 = 3(x - 1)\]
   \[y + 1 = 3(x + 1)\]
   \[y + 4 = 3(x + 1)\]
   \[y - 8 = 3(x - 2)\]

50. **CCSS CRITIQUE** Juana thinks that \(f(x)\) and \(g(x)\) have the same slope but different intercepts. Sabrina thinks that \(f(x)\) and \(g(x)\) describe the same line. Is either of them correct? Explain your reasoning.
   
   "The graph of \(g(x)\) is the line that passes through \((3, -7)\) and \((-6, 4)\)."

51. OPEN ENDED Describe a real-life scenario that has a constant rate of change and a value of \(y\) for a particular value of \(x\). Represent this situation using an equation in point-slope form, an equation in standard form, and an equation in slope-intercept form.

52. REASONING Write an equation for the line that passes through \((-4, 8)\) and \((3, -7)\). What is the slope? Where does the line intersect the \(x\)-axis? the \(y\)-axis?

53. CHALLENGE Write an equation in point-slope form for the line that passes through the points \((f, g)\) and \((h, j)\).

54. WRITING IN MATH Why do we represent linear equations in more than one form?
55. Which statement is most strongly supported by the graph?

A You have $100 and spend $5 weekly.
B You have $100 and save $5 weekly.
C You need $100 for a new CD player and save $5 weekly.
D You need $100 for a new CD player and spend $5 weekly.

56. SHORT RESPONSE A store offers customers a $5 gift certificate for every $75 they spend. How much would a customer have to spend to earn $35 worth of gift certificates?

57. GEOMETRY Which triangle is similar to △ABC?

A B C D

58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?

A 4 C 10
B 7 D 22

Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

59. (4, 2), (−2, −4)
60. (3, −2), (6, 4)
61. (−1, 3), (2, −3)
62. (2, −2), (3, 2)
63. (7, −2), (−4, −2)
64. (0, 5), (−3, 5)

Write an equation in slope-intercept form of the line with the given slope and y-intercept. (Lesson 4-1)

65. slope: −2, y-intercept: 6
66. slope: 3, y-intercept: −5
67. slope: 1/2, y-intercept: 3
68. slope: −3/5, y-intercept: 12
69. slope: 0, y-intercept: 3
70. slope: −1, y-intercept: 0

71. THEATER The Coral Gables Actors’ Playhouse has 7 rows of seats in the orchestra section. The number of seats in the rows forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Skills Review

Solve each equation or formula for the variable specified.

72. \( y = mx + b \), for \( m \)
73. \( v = r + at \), for \( a \)
74. \( km + 5x = 6y \), for \( m \)
75. \( 4b - 5 = -t \), for \( b \)
Parallel Lines

Lines in the same plane that do not intersect are called parallel lines. Nonvertical parallel lines have the same slope.

You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

Example 1 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through \((-3, 5)\) and is parallel to the graph of \(y = 2x - 4\).

Step 1 The slope of the line with equation \(y = 2x - 4\) is 2. The line parallel to \(y = 2x - 4\) has the same slope, 2.

Step 2 Find the equation in slope-intercept form.

\[
\begin{align*}
\quad y - y_1 &= m(x - x_1) & \text{Point-slope form} \\
\quad y - 5 &= 2[x - (-3)] & \text{Replace } m \text{ with } 2 \text{ and } (x_1, y_1) \text{ with } (-3, 5). \\
\quad y - 5 &= 2(x + 3) & \text{Simplify.} \\
\quad y - 5 &= 2x + 6 & \text{Distributive Property} \\
\quad y - 5 + 5 &= 2x + 6 + 5 & \text{Add 5 to each side.} \\
\quad y &= 2x + 11 & \text{Write the equation in slope-intercept form.}
\end{align*}
\]

Guided Practice

1. Write an equation in point-slope form for the line that passes through \((4, -1)\) and is parallel to the graph of \(y = \frac{1}{4}x + 7\).
2 Perpendicular Lines  Lines that intersect at right angles are called **perpendicular lines**. The slopes of nonvertical perpendicular lines are opposite reciprocals. That is, if the slope of a line is $\frac{a}{b}$, the slope of the line perpendicular to it is $-\frac{b}{a}$.

You can use slope to determine whether two lines are perpendicular.

**Real-World Example 2**  **Slopes of Perpendicular Lines**

**DESIGN**  The outline of a company's new logo is shown on a coordinate plane.

a. Is $\angle DFE$ a right angle in the logo?

If $\overline{BE}$ and $\overline{AD}$ are perpendicular, then $\angle DFE$ is a right angle. Find the slopes of $\overline{BE}$ and $\overline{AD}$.

slope of $\overline{BE}$: $m = \frac{1 - 3}{7 - 2}$ or $\frac{2}{5}$

slope of $\overline{AD}$: $m = \frac{6 - 1}{4 - 2}$ or $\frac{5}{2}$

The line segments are perpendicular because $\frac{2}{5} \times \frac{5}{2} = -1$. Therefore, $\angle DFE$ is a right angle.

b. Is each pair of opposite sides parallel?

If a pair of opposite sides are parallel, then they have the same slope.

slope of $\overline{AC}$: $m = \frac{6 - 1}{2 - 2}$ or undefined

Since $\overline{AC}$ and $\overline{GE}$ are both parallel to the $y$-axis, they are vertical and are therefore parallel.

slope of $\overline{CG}$: $m = \frac{6 - 6}{7 - 2}$ or 0

Since $\overline{CG}$ and $\overline{AE}$ are both parallel to the $x$-axis, they are horizontal and are therefore parallel.

**Guided Practice**

2. **CONSTRUCTION**  On the plans for a treehouse, a beam represented by $\overline{QR}$ has endpoints $Q(-6, 2)$ and $R(-1, 8)$. A connecting beam represented by $\overline{ST}$ has endpoints $S(-3, 6)$ and $T(-8, 5)$. Are the beams perpendicular? Explain.

You can determine whether the graphs of two linear equations are parallel or perpendicular by comparing the slopes of the lines.
Example 3 Parallel or Perpendicular Lines

Determine whether the graphs of \( y = 5 \), \( x = 3 \), and \( y = -2x + 1 \) are parallel or perpendicular. Explain.

Graph each line on a coordinate plane.

From the graph, you can see that \( y = 5 \) is parallel to the \( x \)-axis and \( x = 3 \) is parallel to the \( y \)-axis. Therefore, they are perpendicular. None of the lines are parallel.

Guided Practice

3. Determine whether the graphs of \( 6x - 2y = -2 \), \( y = 3x - 4 \), and \( y = 4 \) are parallel or perpendicular. Explain.

You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

Example 4 Perpendicular Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through \((-4, 6)\) and is perpendicular to the graph of \(2x + 3y = 12\).

Step 1 Find the slope of the given line by solving the equation for \( y \).

\[
2x + 3y = 12 \quad \text{Original equation}
\]

\[
2x - 2x + 3y = -2x + 12 \quad \text{Subtract } 2x \text{ from each side.}
\]

\[
3y = -2x + 12 \quad \text{Simplify.}
\]

\[
\frac{3y}{3} = \frac{-2x + 12}{3} \quad \text{Divide each side by } 3.
\]

\[
y = \frac{-2}{3}x + 4 \quad \text{Simplify.}
\]

The slope is \(-\frac{2}{3}\).

Step 2 The slope of the perpendicular line is the opposite reciprocal of \(-\frac{2}{3}\) or \(\frac{3}{2}\). Find the equation of the perpendicular line.

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form}
\]

\[
y - 6 = \frac{3}{2}(x - (-4)) \quad (x_1, y_1) = (-4, 6) \text{ and } m = \frac{3}{2}
\]

\[
y - 6 = \frac{3}{2}(x + 4) \quad \text{Simplify.}
\]

\[
y - 6 = \frac{3}{2}x + 6 \quad \text{Distributive Property}
\]

\[
y - 6 + 6 = \frac{3}{2}x + 6 + 6 \quad \text{Add } 6 \text{ to each side.}
\]

\[
y = \frac{3}{2}x + 12 \quad \text{Simplify.}
\]

Guided Practice

4. Write an equation in slope-intercept form for the line that passes through \((4, 7)\) and is perpendicular to the graph of \(y = \frac{2}{3}x - 1\).
**Concept Summary** Parallel and Perpendicular Lines

<table>
<thead>
<tr>
<th>Parallel Lines</th>
<th>Perpendicular Lines</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Words</strong></td>
<td>Two nonvertical lines are parallel if they have the same slope.</td>
</tr>
<tr>
<td><strong>Symbols</strong></td>
<td>( \overrightarrow{AB} \parallel \overrightarrow{CD} )</td>
</tr>
<tr>
<td><strong>Models</strong></td>
<td><img src="image1" alt="Parallel Lines Diagram" /></td>
</tr>
</tbody>
</table>

**Check Your Understanding**

**Example 1**
Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

1. \((-1, 2), y = \frac{1}{2}x - 3\)  
2. \((0, 4), y = -4x + 5\)

**Example 2**
3. **GARDENS** A garden is in the shape of a quadrilateral with vertices \(A(-2, 1), B(3, -3), C(5, 7), \) and \(D(-3, 4)\). Two paths represented by \(\overrightarrow{AC} \) and \(\overrightarrow{BD} \) cut across the garden. Are the paths perpendicular? Explain.

**Example 3**
Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

5. \(y = -2x, 2y = x, 4y = 2x + 4\)  
6. \(y = \frac{1}{2}x, 3y = x, y = -\frac{1}{2}x\)

**Example 4**
Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

7. \((-2, 3), y = -\frac{1}{2}x - 4\)  
8. \((-1, 4), y = 3x + 5\)

9. \((2, 3), 2x + 3y = 4\)  
10. \((3, 6), 3x - 4y = -2\)
Example 1  Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of the given equation.

11. (3, −2), \( y = x + 4 \)  
12. (4, −3), \( y = 3x - 5 \)  
13. (0, 2), \( y = -5x + 8 \)  
14. (−4, 2), \( y = -\frac{1}{2}x + 6 \)  
15. (−2, 3), \( y = -\frac{3}{4}x + 4 \)  
16. (9, 12), \( y = 13x - 4 \)

Example 2  

17. GEOMETRY A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Is \( ABCD \) a trapezoid? Explain your reasoning.

18. GEOMETRY \( CDEF \) is a kite. Are the diagonals of the kite perpendicular? Explain your reasoning.

19. Determine whether the graphs of \( y = -6x + 4 \) and \( y = \frac{1}{6}x \) are perpendicular. Explain.

20. MAPS On a map, Elmwood Drive passes through \( R(4, -11) \) and \( S(0, -9) \), and Taylor Road passes through \( J(6, -2) \) and \( K(4, -5) \). If they are straight lines, are the two streets perpendicular? Explain.

Example 3  

21. 2\( x - 8y = -24 \), 4\( x + y = -2 \), \( x - 4y = 4 \)

22. 3\( x - 9y = 9 \), 3\( y = x + 12 \), 2\( x - 6y = 12 \)

Example 4  

23. (−3, −2), \( y = -2x + 4 \)  
24. (−5, 2), \( y = \frac{1}{2}x - 3 \)  
25. (−4, 5), \( y = \frac{1}{3}x + 6 \)  
26. (2, 6), \( y = -\frac{1}{4}x + 3 \)  
27. (3, 8), \( y = 5x - 3 \)  
28. (4, −2), \( y = 3x + 5 \)

Write an equation in slope-intercept form for the line that is perpendicular to the graph of the equation.

29. \( y = -\frac{1}{2}x - 4 \)  
30. \( y = \frac{2}{3}x - 6 \)  
31. \( y = 5x + 3 \)

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the \( x \)-intercept of that line.

32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of \( 3x + 2y = 8 \) and passes through the \( y \)-intercept of that line.

Determine whether the graphs of each pair of equations are \textit{parallel}, \textit{perpendicular}, or \textit{neither}.

33. \( y = 4x + 3 \)  
34. \( y = -2x \)  
35. \( 3x + 5y = 10 \)  
36. \( -3x + 4y = 8 \)  
37. \( 2x + 5y = 15 \)  
38. \( 2x + 7y = -35 \)  
39. \( 4x + 14y = -42 \)
39. Write an equation of the line that is parallel to the graph of \( y = 7x - 3 \) and passes through the origin.

40. **EXCAVATION** Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from \((-5, 8)\) to \((10, -1)\) and a second bone lies from \((-10, -3)\) to \((-5, -6)\), are the bones parallel? Explain.

41. **ARCHAEOLOGY** In the ruins of an ancient civilization, an archaeologist found pottery at \((2, 6)\) and hair accessories at \((4, -1)\). A pole is found with one end at \((7, 10)\) and the other end at \((14, 12)\). Is the pole perpendicular to the line through the pottery and the hair accessories? Explain.

42. **GRAPHICS** To create a design on a computer, Andeana must enter the coordinates for points on the design. One line segment she drew has endpoints of \((-2, 1)\) and \((4, 3)\). The other coordinates that Andeana entered are \((2, -7)\) and \((8, -3)\). Could these points be the vertices of a rectangle? Explain.

43. **MULTIPLE REPRESENTATIONS** In this problem, you will explore parallel and perpendicular lines.

   a. **Graphical** Graph the points \(A(-3, 3)\), \(B(3, 5)\), and \(C(-4, 0)\) on a coordinate plane.

   b. **Analytical** Determine the coordinates of a fourth point \(D\) that would form a parallelogram. Explain your reasoning.

   c. **Analytical** What is the minimum number of points that could be moved to make the parallelogram a rectangle? Describe which points should be moved, and explain why.

**H.O.T. Problems** Use Higher-Order Thinking Skills

44. **CHALLENGE** If the line through \((-2, 4)\) and \((5, d)\) is parallel to the graph of \(y = 3x + 4\), what is the value of \(d\)?

45. **REASONING** Which key features of the graphs of two parallel lines are the same, and which are different? Which key features of the graphs of two perpendicular lines are the same, and which are different?

46. **OPEN ENDED** Graph a line that is parallel and a line that is perpendicular to \(y = 2x - 1\).

**Example 3**

47. **CRITIQUE** Carmen and Chase are finding an equation of the line that is perpendicular to the graph of \(y = \frac{1}{3}x + 2\) and passes through the point \((-3, 5)\). Is either of them correct? Explain your reasoning.

**Carmen**

\[
\begin{align*}
  y - 5 &= -3[x - (-3)] \\
  y - 5 &= -3(x + 3) \\
  y &= -3x - 9 + 5 \\
  y &= -3x - 4
\end{align*}
\]

**Chase**

\[
\begin{align*}
  y - 5 &= 3[x - (-3)] \\
  y - 5 &= 3(x + 3) \\
  y &= 3x + 9 + 5 \\
  y &= 3x + 14
\end{align*}
\]

48. **WRITING IN MATH** Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation for the graph that is parallel and an equation for the graph that is perpendicular to the line shown. Explain your reasoning.
49. Which of the following is an algebraic translation of the following phrase?
5 less than the quotient of a number and 8
A $5 - \frac{n}{8}$  
B $\frac{n}{8} - 5$  
C $5 - \frac{8}{n}$  
D $\frac{8}{n} - 5$

50. A line through which two points would be parallel to a line with a slope of $\frac{3}{4}$?  
F (0, 5) and (−4, 2)  
G (0, 2) and (−4, 1)  
H (0, 0) and (0, −2)  
J (0, −2) and (−4, −2)

51. Which equation best fits the data in the table?
A $y = x + 4$  
B $y = 2x + 3$  
C $y = 7$  
D $y = 4x - 5$

52. SHORT RESPONSE  
Tyler is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gallons. How many total hours will it take to completely fill the pool?

Spiral Review

Write each equation in standard form. (Lesson 4-3)
53. $y - 13 = 4(x - 2)$  
54. $y - 5 = -2(x + 2)$  
55. $y + 3 = -5(x + 1)$  
56. $y + 7 = \frac{1}{2}(x + 2)$  
57. $y - 1 = \frac{5}{6}(x - 4)$  
58. $y - 2 = -\frac{2}{5}(x - 8)$

59. CANOE RENTAL  
Latanya and her friends rented a canoe for 3 hours and paid a total of $45. (Lesson 4-2)

a. Write a linear equation to find the total cost $C$ of renting the canoe for $h$ hours.

b. How much would it cost to rent the canoe for 8 hours?

Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)
60. (5, −2), $m = 3$  
61. (−5, 4), $m = −5$  
62. (3, 0), $m = −2$  
63. (3, 5), $m = 2$  
64. (−3, −1), $m = −3$  
65. (−2, 4), $m = −5$

Simplify each expression. If not possible, write simplified. (Lesson 1-4)
66. $13m + m$  
67. $14a^2 + 13b^2 + 27$  
68. $3(x + 2x)$

69. FINANCIAL LITERACY  
At a Farmers’ Market, merchants can rent a small table for $5.00 and a large table for $8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

a. Write an algebraic expression to show the total amount of money collected.

b. Evaluate the expression.

Skills Review

Express each relation as a graph. Then determine the domain and range.
70. {(3, 8), (3, 7), (2, −9), (1, −9), (−5, −3)}  
71. {(3, 4), (4, 3), (2, 2), (5, −4), (−4, 5)}  
72. {(0, 2), (−5, 1), (0, 6), (−1, 9), (−4, −5)}  
73. {((−7, 6), (−3, −4), (4, −5), (−2, 6), (−3, 2)}
Write an equation in slope-intercept form for each graph shown. (Lesson 4-1)

1. \( y = \frac{1}{2}x + 2 \)

2. \( y = 2x + 3 \)

Graph each equation. (Lesson 4-1)

3. \( y = 2x + 3 \)

4. \( y = \frac{1}{3}x - 2 \)

5. **BOATS** Write an equation in slope-intercept form for the total rental cost \( C \) for a pontoon boat used for \( t \) hours. (Lesson 4-1)

Write an equation of the line with the given conditions. (Lesson 4-2)

6. \((2, 5); \) slope 3

7. \((-3, -1), \) slope \( \frac{1}{2} \)

8. \((-3, 4), (1, 12) \)

9. \((-1, 6), (2, 4) \)

10. \((2, 1), \) slope 0

11. **MULTIPLE CHOICE** Write an equation of the line that passes through the point \((0, 0)\) and has slope \(-4\). (Lesson 4-2)

   - A \( y = x - 4 \)
   - B \( y = x + 4 \)
   - C \( y = -4x \)
   - D \( y = 4 - x \)

   Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

12. \((1, 4), m = 6 \)

13. \((-2, -1), m = -3 \)

14. Write an equation in point-slope form for the line that passes through the point \((8, 3), m = -2\). (Lesson 4-3)

15. Write \( y + 3 = \frac{1}{2}(x - 5) \) in standard form. (Lesson 4-3)

16. Write \( y + 4 = -7(x - 3) \) in slope-intercept form. (Lesson 4-3)

Write each equation in standard form. (Lesson 4-3)

17. \( y - 5 = -2(x - 3) \)

18. \( y + 4 = \frac{2}{3}(x - 3) \)

Write each equation in slope-intercept form. (Lesson 4-3)

19. \( y - 3 = 4(x + 3) \)

20. \( y + 1 = \frac{1}{2}(x - 8) \)

21. **MULTIPLE CHOICE** Determine whether the graphs of the pair of equations are parallel, perpendicular, or neither. (Lesson 4-4)

   \[ y = -6x + 8 \]

   \[ 3x + \frac{1}{2}y = -3 \]

   - F parallel
   - G perpendicular
   - H neither
   - J not enough information

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

22. \((3, -4); y = -\frac{1}{3}x - 5 \)

23. \((0, -3); y = -2x + 4 \)

24. \((-4, -5); -4x + 5y = -6 \)

25. \((-1, -4); -x - 2y = 0 \)
Investigate Relationships Using Scatter Plots

Data with two variables are called **bivariate data**. A **scatter plot** shows the relationship between a set of data with two variables, graphed as ordered pairs on a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

### Concept Summary Scatter Plots

**Positive Correlation**
- As \( x \) increases, \( y \) increases
- Linear interpolation

**Negative Correlation**
- As \( x \) decreases, \( y \) decreases
- Linear interpolation

**No Correlation**
- \( x \) and \( y \) are not related

### Real-World Example 1 Evaluate a Correlation

**WAGES**
Determine whether the graph shows a **positive**, **negative**, or **no** correlation. If there is a positive or negative correlation, describe its meaning in the situation.

The graph shows a positive correlation. As the number of hours worked increases, the wages usually increase.

### Guided Practice

1. Refer to the graph on international travel. Determine whether the graph shows a **positive**, **negative**, or **no** correlation. If there is a positive or negative correlation, describe its meaning.
Use Lines of Fit

Scatter plots can show whether there is a trend in a set of data. When the data points all lie close to a line, a line of fit or trend line can model the trend.

Key Concept Using a Linear Function to Model Data

Step 1 Make a scatter plot. Determine whether any relationship exists in the data.

Step 2 Draw a line that seems to pass close to most of the data points.

Step 3 Use two points on the line of fit to write an equation for the line.

Step 4 Use the line of fit to make predictions.

Real-World Example 2 Write a Line of Fit

ROLLER COASTERS The table shows the largest vertical drops of nine roller coasters in the United States and the number of years after 1988 that they were opened. Identify the independent and the dependent variables. Is there a relationship in the data? If so, predict the vertical drop in a roller coaster built 30 years after 1988.

<table>
<thead>
<tr>
<th>Years Since 1988</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>12</th>
<th>12</th>
<th>12</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical Drop (ft)</td>
<td>151</td>
<td>155</td>
<td>225</td>
<td>230</td>
<td>306</td>
<td>300</td>
<td>255</td>
<td>255</td>
<td>400</td>
</tr>
</tbody>
</table>

Source: Ultimate Roller Coaster

Step 1 Make a scatter plot.

The independent variable is the year, and the dependent variable is the vertical drop. As the number of years increases, the vertical drop of roller coasters increases. There is a positive correlation between the two variables.

Step 2 Draw a line of fit.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown.

Step 3 Write the slope-intercept form of an equation for the line of fit.

The line of fit passes close to (2, 150) and the data point (12, 300).

Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{300 - 150}{12 - 2} \]

\[ = 15 \]

Use \( m = 15 \) and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

\[ y - y_1 = m(x - x_1) \]

\[ y - 150 = 15(x - 2) \]

\[ y - 150 = 15x - 30 \]

\[ y = 15x + 120 \]

A slope of 15 means that the vertical drops increased an average of 15 feet per year. To predict the vertical drop of a roller coaster built 30 years after 1988, substitute 30 for \( x \) in the equation. The vertical drop is 15(30) + 120 or 570 feet.

Real-World Link

The Kingda Ka roller coaster at Six Flags Great Adventure in Jackson, New Jersey, has broken three records: tallest roller coaster at 456 feet, fastest at 128 miles per hour, and largest vertical drop of 418 feet.

Source: Ultimate Roller Coaster
Guided Practice

2. MUSIC The table shows the dollar value in millions for the sales of CDs for the year. Make a scatter plot and determine what relationship exists, if any.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>13,215</td>
<td>12,909</td>
<td>12,044</td>
<td>11,233</td>
<td>11,447</td>
<td>10,520</td>
<td>9,373</td>
<td>7,452</td>
<td>5,471</td>
</tr>
</tbody>
</table>

In Lesson 4-2, you learned that linear extrapolation is used to predict values outside the range of the data. You can also use a linear equation to predict values inside the range of the data. This is called **linear interpolation**.

Real-World Example 3 Use Interpolation or Extrapolation

**TRAVEL** Use the scatter plot to find the approximate number of United States travelers to international countries in 1996.

**Step 1** Draw a line of fit. The line should be as close to as many points as possible.

**Step 2** Write the slope-intercept form of the equation. The line of fit passes through (0, 44,623) and (18, 63,554).

Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{63,554 - 44,623}{18 - 0} \]

\[ = \frac{18,931}{18} \]

Use \( m = \frac{18,931}{18} \) and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

\[ y - y_1 = m(x - x_1) \]

\[ y - 44,623 = \frac{18,931}{18}(x - 0) \]

\[ y - 44,623 = \frac{18,931}{18}x \]

\[ y = \frac{18,931}{18}x + 44,623 \]

**Step 3** Evaluate the function for \( x = 1996 - 1990 \) or 6.

\[ y = \frac{18,931}{18}x + 44,623 \]

\[ = \frac{18,931}{18}(6) + 44,623 \]

\[ = 6310\frac{1}{3} + 44,623 \text{ or } 50,933\frac{1}{3} \]

In 1996, there were approximately 50,933 thousand or 50,933,000 people who traveled from the United States to international countries.

Guided Practice

3. MUSIC Use the equation for the line of fit for the data in Guided Practice 2 to estimate CD sales in 2015.
Example 1  Determine whether each graph shows a positive, negative, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

1. \[ \text{Free Throws Made (\%)} \]
   \[ \text{Practice Minutes Per Day} \]

2. \[ \text{Lemonade Sales ($)} \]
   \[ \text{Temperature (°F)} \]

Example 2  3. SENSE-MAKING  The table shows the median age of females when they were first married.
   a. Make a scatter plot and determine what relationship exists, if any, in the data. Identify the independent and the dependent variables.
   b. Draw a line of fit for the scatter plot.
   c. Write an equation in slope-intercept form for the line of fit.
   
Example 3  d. Predict what the median age of females when they are first married will be in 2016.
   e. Do you think the equation can give a reasonable estimate for the year 2056? Explain.

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>24.8</td>
</tr>
<tr>
<td>1997</td>
<td>25.0</td>
</tr>
<tr>
<td>1998</td>
<td>25.0</td>
</tr>
<tr>
<td>1999</td>
<td>25.1</td>
</tr>
<tr>
<td>2000</td>
<td>25.1</td>
</tr>
<tr>
<td>2001</td>
<td>25.1</td>
</tr>
<tr>
<td>2002</td>
<td>25.3</td>
</tr>
<tr>
<td>2003</td>
<td>25.3</td>
</tr>
<tr>
<td>2004</td>
<td>25.3</td>
</tr>
<tr>
<td>2005</td>
<td>25.5</td>
</tr>
<tr>
<td>2006</td>
<td>25.9</td>
</tr>
</tbody>
</table>

Source: U.S. Bureau of Census

Practice and Problem Solving

Example 1  Determine whether each graph shows a positive, negative, or no correlation. If there is a positive or negative correlation, describe its meaning in the situation.

4. \[ \text{Game Tickets at the Fair} \]

5. \[ \text{NBA 3-Point Percentage} \]
8. **MILK** Refer to the scatter plot of gallons of milk consumption per person for selected years.
   
   a. Use the points (2, 21.75) and (4, 21) to write the slope-intercept form of an equation for the line of fit.
   
   b. Predict the milk consumption in 2020.
   
   c. Predict in what year milk consumption will be 10 gallons.
   
   d. Is it reasonable to use the equation to estimate the consumption of milk for any year? Explain.

9. **FOOTBALL** Use the scatter plot.
   
   a. Use the points (5, 71,205) and (9, 68,611) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.
   
   b. Predict the average attendance at a game in 2020.
   
   c. Can you use the equation to make a decision about the average attendance in any given year in the future? Explain.

10. **SENSE-MAKING** The Body Mass Index (BMI) is a measure of body fat using height and weight. The heights and weights of twelve men with normal BMI are given in the table at the right.
    
    a. Make a scatter plot comparing the height in inches to the weight in pounds.
    
    b. Draw a line of fit for the data.
    
    c. Write the slope-intercept form of an equation for the line of fit.
    
    d. Predict the normal weight for a man who is 84 inches tall.
    
    e. A man’s weight is 188 pounds. Use the equation of the line of fit to predict the height of the man.
GEYSERS  The time to the next eruption of Old Faithful can be predicted by using the duration of the current eruption.

<table>
<thead>
<tr>
<th>Duration (min)</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval (min)</td>
<td>48</td>
<td>55</td>
<td>70</td>
<td>72</td>
<td>74</td>
<td>82</td>
<td>93</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Identify the independent and the dependent variables. Make a scatter plot and determine what relationship, if any, exists in the data. Draw a line of fit for the scatter plot.

b. Let \( x \) represent the duration of the previous interval. Let \( y \) represent the time between eruptions. Write the slope-intercept form of the equation for the line of fit. Predict the interval after a 7.5-minute eruption.

c. Make a critical judgment about using the equation to predict the duration of the next eruption. Would the equation be a useful model?

12. COLLECT DATA  Use a tape measure to measure both the foot size and the height in inches of ten individuals.

a. Record your data in a table.

b. Make a scatter plot and draw a line of fit for the data.

c. Write an equation for the line of fit.

d. Make a conjecture about the relationship between foot size and height.

H.O.T. Problems  Use Higher-Order Thinking Skills

13. OPEN ENDED  Describe a real-life situation that can be modeled using a scatter plot. Decide whether there is a positive, negative, or no correlation. Explain what this correlation means.

14. WHICH ONE DOESN’T BELONG?  Analyze the following situations and determine which one does not belong.

- hours worked and amount of money earned
- height of an athlete and favorite color
- seedlings that grow an average of 2 centimeters each week
- number of photos stored on a camera and capacity of camera

15. **CSS** ARGUMENTS  Determine which line of fit is better for the scatter plot. Explain your reasoning.

16. REASONING  What can make a scatter plot and line of fit more useful for accurate predictions? Does an accurate line of fit always predict what will happen in the future? Explain.

17. WRITING IN MATH  Make a scatter plot that shows the height of a person and age. Explain how you could use the scatter plot to predict the age of a person given his or her height. How can the information from a scatter plot be used to identify trends and make decisions?
18. Which equation best describes the relationship between the values of \( x \) and \( y \) in the table?

A \( y = x - 5 \)
B \( y = 2x - 5 \)
C \( y = 3x - 7 \)
D \( y = 4x - 7 \)

![Table]

19. **STATISTICS** Mr. Hernandez collected data on the heights and average stride lengths of a random sample of high school students. He then made a scatter plot. What kind of correlation did he most likely see?

F positive  
H negative  
G constant  
J no

20. **GEOMETRY** Mrs. Aguilar’s rectangular bedroom measures 13 feet by 11 feet. She wants to purchase carpet for the bedroom that costs $2.95 per square foot, including tax. How much will the carpet cost?

A $70.80  
B $141.60  
C $145.95  
D $421.85

21. **SHORT RESPONSE** Nikia bought a one-month membership to a fitness center for $35. Each time she goes, she rents a locker for $0.25. If she spent $40.50 at the fitness center last month, how many days did she go?

22. Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither. (Lesson 4-4)

22. \( y = -2x + 11 \)
   \( y + 2x = 23 \)

24. \( y = -5x \)
   \( y = 5x - 18 \)

23. \( 3y = 2x + 14 \)
   \( 2x + 3y = 2 \)

25. \( y = 3x + 2 \)
   \( y = -\frac{1}{3}x - 2 \)

26. Write each equation in standard form. (Lesson 4-3)

26. \( y - 13 = 4(x - 2) \)

27. \( y - 5 = -2(x + 2) \)

29. \( y + 7 = \frac{1}{2}(x + 2) \)

30. \( y - 1 = \frac{5}{6}(x - 4) \)

28. \( y + 3 = -5(x + 1) \)

31. \( y - 2 = -\frac{2}{5}(x - 8) \)

Graph each equation. (Lesson 4-1)

32. \( y = 2x + 3 \)

33. \( 4x + y = -1 \)

34. \( 3x + 4y = 7 \)

35. Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35. \((3, 4), (10, 8)\)

36. \((-4, 7), (3, 5)\)

37. \((3, 7), (-2, 4)\)

38. \((-3, 2), (-3, 4)\)

39. \((-2, -6), (-1, 10)\)

40. \((1, -5), (-3, -5)\)

41. **DRIVING** Latisha drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles? (Lesson 2-6)

42. **Skills Review** Express each relation as a graph. Then determine the domain and range.

42. \\{(4, 5), (5, 4), (-2, -2), (4, -5), (-5, 4)\}

43. \\{(7, 6), (3, 4), (4, 5), (-2, 6), (-3, 2)\}

| connectEd.mcgraw-hill.com | 253 |
You may be considering attending a college or technical school in the future. What factors cause tuition to rise—increased building costs, higher employee salaries, or the amount of bottled water consumed?

Let’s see how bottled water and college tuition are related. The table shows the average college tuition and fees for public colleges and the per person U.S. consumption of bottled water per year for 2003 through 2007.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Consumed (gallons)</td>
<td>21.6</td>
<td>23.2</td>
<td>25.4</td>
<td>27.6</td>
<td>29.3</td>
</tr>
<tr>
<td>Tuition ($)</td>
<td>4645</td>
<td>5126</td>
<td>5492</td>
<td>5804</td>
<td>6191</td>
</tr>
</tbody>
</table>

Source: Beverage Marketing Corporation and College Board

**Activity  Correlation and Causation**

Follow the steps to learn about correlation and causation.

**Step 1** Graph the ordered pairs (gallons, tuition) to create a scatter plot. For example, one ordered pair is (21.6, 4645). Describe the graph.

**Step 2** Is the correlation *positive* or *negative*? Explain.

**Step 3** Do you think drinking more bottled water *causes* college tuition costs to rise? Explain.

**Step 4** Causation occurs when a change in one variable produces a change in another variable. Correlation can be observed between many variables, but causation can only be determined from data collected from a controlled experiment. Describe an experiment that could illustrate causation.

**Exercises**

For each exercise, determine whether each situation illustrates *correlation* or *causation*. Explain your reasoning, including other factors that might be involved.

1. A survey showed that sleeping with the light on was positively correlated to nearsightedness.
2. A controlled experiment showed a positive correlation between the number of cigarettes smoked and the probability of developing lung cancer.
3. A random sample of students found that owning a cell phone had a negative correlation with riding the bus to school.
4. A controlled experiment showed a positive correlation between the number of hours using headphones when listening to music and the level of hearing loss.
5. DeQuan read in the newspaper that shark attacks are positively correlated with monthly ice cream sales.
Regression and Median-Fit Lines

New Vocabulary

- best-fit line
- linear regression
- correlation coefficient
- residual
- median-fit line

Best-Fit Lines

You have learned how to find and write equations for lines of fit by hand. Many calculators use complex algorithms that find a more precise line of fit called the **best-fit line**. One algorithm is called **linear regression**.

Your calculator may also compute a number called the **correlation coefficient**. This number will tell you if your correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or -1, the more closely the equation models the data.

Real-World Example 1

MOVIES

The table shows the amount of money made by movies in the United States. Use a graphing calculator to write an equation for the best-fit line for that data.

<table>
<thead>
<tr>
<th>Year</th>
<th>Income ($ billion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>7.48</td>
</tr>
<tr>
<td>2001</td>
<td>8.13</td>
</tr>
<tr>
<td>2002</td>
<td>9.19</td>
</tr>
<tr>
<td>2003</td>
<td>9.35</td>
</tr>
<tr>
<td>2004</td>
<td>9.27</td>
</tr>
<tr>
<td>2005</td>
<td>8.95</td>
</tr>
<tr>
<td>2006</td>
<td>9.25</td>
</tr>
<tr>
<td>2007</td>
<td>9.65</td>
</tr>
<tr>
<td>2008</td>
<td>9.85</td>
</tr>
<tr>
<td>2009</td>
<td>10.21</td>
</tr>
</tbody>
</table>

Before you begin, make sure that your Diagnostic setting is on. You can find this under the **CATALOG** menu. Press **D** and then scroll down and click **DiagnosticOn**. Then press **ENTER**.

**Step 1**

Enter the data by pressing **STAT** and selecting the **Edit** option. Let the year 2000 be represented by 0. Enter the years since 2000 into List 1 (L1). These will represent the x-values. Enter the income ($ billion) into List 2 (L2). These will represent the y-values.

**Step 2**

Perform the regression by pressing **STAT** and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press **ENTER** twice.
Step 3 Write the equation of the regression line by rounding the $a$ and $b$ values on the screen. The form that we chose for the regression was $ax + b$, so the equation is $y = 0.23x + 8.09$. The correlation coefficient is about 0.8755, which means that the equation models the data fairly well.

Guided Practice

Write an equation of the best-fit line for the data in each table. Name the correlation coefficient. Round to the nearest ten-thousandth. Let $x$ be the number of years since 2003.

1A. HOCKEY The table shows the number of goals of leading scorers for the Mustang Girls Hockey Team.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goals</td>
<td>30</td>
<td>23</td>
<td>41</td>
<td>35</td>
<td>31</td>
<td>43</td>
<td>33</td>
<td>45</td>
</tr>
</tbody>
</table>

1B. HOCKEY The table gives the number of goals scored by the team each season.

<table>
<thead>
<tr>
<th>Year</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goals</td>
<td>63</td>
<td>44</td>
<td>55</td>
<td>63</td>
<td>81</td>
<td>85</td>
<td>93</td>
<td>84</td>
</tr>
</tbody>
</table>

We know that not all of the points will lie on the best-fit line. The difference between an observed $y$-value and its predicted $y$-value (found on the best-fit line) is called a residual. Residuals measure how much the data deviate from the regression line. When residuals are plotted on a scatter plot they can help to assess how well the best-fit line describes the data. If the best-fit line is a good fit, there is no pattern in the residual plot.

Real-World Example 2 Graph and Analyze a Residual Plot

HOCKEY Graph and analyze the residual plot for the data for Guided Practice 1A. Determine if the best-fit line models the data well.

After calculating the best-fit line in Guided Practice 1A, you can obtain the residual plot of the data. Turn on Plot2 under the STAT PLOT menu and choose L3:.. Use L1 for the Xlist and RESID for the Ylist. You can obtain RESID by pressing 2nd [STAT] and selecting RESID from the list of names. Graph the scatter plot of the residuals by pressing ZOOM and choosing ZoomStat.

The residuals appear to be randomly scattered and centered about the line $y = 0$. Thus, the best-fit line seems to model the data well.

Real-World Link In 1994, Minnesota became the first state to sanction girls' ice hockey as a high school varsity sport.

Source: ESPNET SportsZone
A residual is positive when the observed value is above the line, negative when the observed value is below the line, and zero when it is on the line. One common measure of goodness of fit is the sum of squared vertical distances from the points to the line. The best-fit line, which is also called the least-squares regression line, minimizes the sum of the squares of those distances.

We can use points on the best-fit line to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called linear interpolation. When we estimate a number outside of the range of the data, it is called linear extrapolation.

Real-World Example 3 Use Interpolation and Extrapolation

PAINTBALL The table shows the points received by the top ten paintball teams at a tournament. Estimate how many points the 20th-ranked team received.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>89</td>
</tr>
<tr>
<td>3</td>
<td>96</td>
</tr>
<tr>
<td>4</td>
<td>99</td>
</tr>
<tr>
<td>5</td>
<td>97</td>
</tr>
<tr>
<td>6</td>
<td>98</td>
</tr>
<tr>
<td>7</td>
<td>78</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
</tr>
<tr>
<td>9</td>
<td>64</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
</tr>
</tbody>
</table>

Write an equation of the best-fit line for the data. Then extrapolate to find the missing value.

Step 1 Enter the data from the table into the lists. Let the ranks be the $x$-values and the scores be the $y$-values. Then graph the scatter plot.

Step 2 Perform the linear regression using the data in the lists. Find the equation of the best-fit line.

The equation is about $y = -3.32x + 105.3$.

Step 3 Graph the best-fit line. Press $Y= \text{VARS}$ and choose Statistics. From the EQ menu, choose $\text{RegEQ}$. Then press $\text{GRAPH}$.

Step 4 Use the graph to predict the points that the 20th-ranked team received. Change the viewing window to include the $x$-value to be evaluated. Press $\text{2nd} \ [\text{CALC}] \ \text{ENTER} \ 20 \ \text{ENTER}$ to find that when $x = 20$, $y \approx 39$.

It is estimated that the 20th ranked team received 39 points.
**Guided Practice**

**ONLINE GAMES** Use linear interpolation to estimate the percent of Americans that play online games for the following ages.

<table>
<thead>
<tr>
<th>Age</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>81</td>
<td>54</td>
<td>37</td>
<td>29</td>
<td>25</td>
</tr>
</tbody>
</table>

Source: Pew Internet & American Life Survey

3A. 35 years  
3B. 18 years

**Median-Fit Lines** A second type of fit line that can be found using a graphing calculator is a *median-fit line*. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.

**Example 4 Median-Fit Line**

**PAINTBALL** Find and graph the equation of a median-fit line for the data in Example 3. Then predict the score of the 15th ranked team.

**Step 1** Reenter the data if it is not in the lists. Clear the Y= list and graph the scatter plot.

**Step 2** To find the median-fit equation, press the STAT key and select the CALC option. Scroll down to the Med-Med option and press ENTER. The value of $a$ is the slope, and the value of $b$ is the $y$-intercept.

The equation for the median-fit line is about $y = -3.71x + 108.26$.

**Step 3** Copy the equation to the Y= list and graph. Use the value option to find the value of $y$ when $x = 15$.

The 15th place team scored about 53 points.

Notice that the equations for the regression line and the median-fit line are very similar.

**Real-World Link**

Paintball is more popular with 12- to 17-year-olds than any other age group. In a recent year, 3,649,000 teens participated in paintball while 2,195,000 18- to 24-year-olds participated.

Source: Statistical Abstract of the United States

**Guided Practice**

4. Use the data from Guided Practice 3 and a median-fit line to estimate the numbers of 18- and 35-year-olds who play online games. Compare these values with the answers from the regression line.
Check Your Understanding

Examples 1, 2  1. POTTERY  A local university is keeping track of the number of art students who use the pottery studio each day.

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>10</td>
<td>15</td>
<td>18</td>
<td>15</td>
<td>13</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

a. Write an equation of the regression line and find the correlation coefficient.
b. Graph the residual plot and determine if the regression line models the data well.

Example 3  2. COMPUTERS  The table below shows the percent of Americans with a broadband connection at home in a recent year. Use linear extrapolation and a regression equation to estimate the percentage of 60-year-olds with broadband at home.

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>40</td>
<td>42</td>
<td>36</td>
<td>35</td>
<td>36</td>
<td>32</td>
</tr>
</tbody>
</table>

Example 4  3. VACATION  The Smiths want to rent a house on the lake that sleeps eight people. The cost of the house per night is based on how close it is to the water.

<table>
<thead>
<tr>
<th>Distance from Lake (mi)</th>
<th>0.0 (houseboat)</th>
<th>0.3</th>
<th>0.5</th>
<th>1.0</th>
<th>1.25</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price/Night ($)</td>
<td>785</td>
<td>325</td>
<td>250</td>
<td>200</td>
<td>150</td>
<td>140</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Find and graph an equation for the median-fit line.
b. What would you estimate is the cost of a rental 1.75 miles from the lake?

Practice and Problem Solving

Example 1  Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

4. SKYSCRAPERS  The table ranks the ten tallest buildings in the world.

<table>
<thead>
<tr>
<th>Rank</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stories</td>
<td>101</td>
<td>88</td>
<td>110</td>
<td>88</td>
<td>88</td>
<td>80</td>
<td>69</td>
<td>102</td>
<td>78</td>
<td>70</td>
</tr>
</tbody>
</table>

5. MUSIC  The table gives the number of annual violin auditions held by a youth symphony each year since 2004. Let \( x \) be the number of years since 2004.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auditions</td>
<td>22</td>
<td>19</td>
<td>25</td>
<td>37</td>
<td>32</td>
<td>35</td>
<td>42</td>
</tr>
</tbody>
</table>

Example 2  6. RETAIL  The table gives the sales at a clothing chain since 2004. Let \( x \) be the number of years since 2004.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (Millions of Dollars)</td>
<td>6.84</td>
<td>7.6</td>
<td>10.9</td>
<td>15.4</td>
<td>17.6</td>
<td>21.2</td>
<td>26.5</td>
</tr>
</tbody>
</table>

a. Write an equation of the regression line.
b. Graph and analyze the residual plot.
MARATHON  The number of entrants in the Boston Marathon every five years since 1975 is shown. Let $x$ be the number of years since 1975.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrants</td>
<td>2395</td>
<td>5417</td>
<td>5594</td>
<td>9412</td>
<td>9416</td>
<td>17,813</td>
<td>20,453</td>
<td>26,735</td>
</tr>
</tbody>
</table>

a. Find an equation for the median-fit line.

b. According to the equation, how many entrants were there in 2003?

8. CAMPING  A campground keeps a record of the number of campsites rented the week of July 4 for several years. Let $x$ be the number of years since 2000.

<table>
<thead>
<tr>
<th>Year</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sites Rented</td>
<td>34</td>
<td>45</td>
<td>42</td>
<td>53</td>
<td>58</td>
<td>47</td>
<td>57</td>
<td>65</td>
<td>59</td>
</tr>
</tbody>
</table>

a. Find an equation for the regression line.

b. Predict the number of campsites that will be rented in 2012.

c. Predict the number of campsites that will be rented in 2020.

9. ICE CREAM  An ice cream company keeps a count of the tubs of chocolate ice cream delivered to each of their stores in a particular area.

a. Find an equation for the median-fit line.

b. Graph the points and the median-fit line.

c. How many tubs would be delivered to a 1500-square-foot store? a 5000-square-foot store?

10. SENSE-MAKING  The prices of the eight top-selling brands of jeans at Jeanie’s Jeans are given in the table below.

<table>
<thead>
<tr>
<th>Sales Rank</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($)</td>
<td>43</td>
<td>44</td>
<td>50</td>
<td>61</td>
<td>64</td>
<td>135</td>
<td>108</td>
<td>78</td>
</tr>
</tbody>
</table>

a. Find the equation for the regression line.

b. According to the equation, what would be the price of a pair of the 12th best-selling brand?

c. Is this a reasonable prediction? Explain.

11. STATE FAIRS  Refer to the beginning of the lesson.

a. Graph a scatter plot of the data, where $x = 1$ represents 2005. Then find and graph the equation for the best-fit line.

b. Graph and analyze the residual plot.

c. Predict the total attendance in 2020.
12. **FIREFIGHTERS** The table shows statistics from the U.S. Fire Administration.

<table>
<thead>
<tr>
<th>Age</th>
<th>Number of Firefighters</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>40,919</td>
</tr>
<tr>
<td>25</td>
<td>245,516</td>
</tr>
<tr>
<td>35</td>
<td>330,516</td>
</tr>
<tr>
<td>45</td>
<td>296,665</td>
</tr>
<tr>
<td>55</td>
<td>167,087</td>
</tr>
<tr>
<td>65</td>
<td>54,559</td>
</tr>
</tbody>
</table>

a. Find an equation for the median-fit line.
b. Graph the points and the median-fit line.
c. Does the median-fit line give you an accurate picture of the number of firefighters? Explain.

13. **ATHLETICS** The table shows the number of participants in high school athletics.

<table>
<thead>
<tr>
<th>Year Since 1970</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Athletes</td>
<td>3,960,932</td>
<td>5,356,913</td>
<td>5,298,671</td>
<td>6,705,223</td>
<td>7,159,904</td>
</tr>
</tbody>
</table>

a. Find an equation for the regression line.
b. According to the equation, how many participated in 1988?

14. **ART** A count was kept on the number of paintings sold at an auction by the year in which they were painted. Let \(x\) be the number of years since 1950.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Paintings Sold</td>
<td>8</td>
<td>5</td>
<td>25</td>
<td>21</td>
<td>9</td>
<td>22</td>
</tr>
</tbody>
</table>

a. Find the equation for the linear regression line.
b. How many paintings were sold that were painted in 1961?
c. Is the linear regression equation an accurate model of the data? Explain why or why not.

**H.O.T. Problems** Use Higher-Order Thinking Skills

15. **ARGUMENTS** Below are the results of the World Superpipe Championships in 2008.

<table>
<thead>
<tr>
<th>Men</th>
<th>Score</th>
<th>Rank</th>
<th>Women</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaun White</td>
<td>93.00</td>
<td>1</td>
<td>Torah Bright</td>
<td>96.67</td>
</tr>
<tr>
<td>Mason Aguirre</td>
<td>90.33</td>
<td>2</td>
<td>Kelly Clark</td>
<td>93.00</td>
</tr>
<tr>
<td>Janne Korpi</td>
<td>85.33</td>
<td>3</td>
<td>Soko Yamaoka</td>
<td>85.00</td>
</tr>
<tr>
<td>Luke Mitrani</td>
<td>85.00</td>
<td>4</td>
<td>Ellery Hollingsworth</td>
<td>79.33</td>
</tr>
<tr>
<td>Keir Dillion</td>
<td>81.33</td>
<td>5</td>
<td>Sophie Rodriguez</td>
<td>71.00</td>
</tr>
</tbody>
</table>

Find an equation of the regression line for each, and graph them on the same coordinate plane. Compare and contrast the men’s and women’s graphs.

16. **REASONING** For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best fit. Could it be used to predict the scores of other students? Explain your reasoning.

17. **OPEN ENDED** For 10 different people, measure their heights and the lengths of their heads from chin to top. Use these data to generate a linear regression equation and a median-fit equation. Make a prediction using both of the equations.

18. **WRITING IN MATH** How are lines of fit and linear regression similar? different?
19. GEOMETRY Sam is putting a border around a poster. $x$ represents the poster’s width, and $y$ represents the poster’s length. Which equation represents how much border Sam will use if he doubles the length and the width?

A $4xy$  
B $(x + y)^4$  
C $4(x + y)$  
D $16(x + y)$

20. SHORT RESPONSE Tatiana wants to run 5 miles at an average pace of 9 minutes per mile. After 4 miles, her average pace is 9 minutes 10 seconds. In how many minutes must she complete the final mile to reach her goal?

21. What is the slope of the line that passes through $(1, 3)$ and $(-3, 1)$?

F $-2$  
H $\frac{1}{2}$  
G $-\frac{1}{2}$  
J $2$

22. What is an equation of the line that passes through $(0, 1)$ and has a slope of 3?

A $y = 3x - 1$  
B $y = 3x - 2$  
C $y = 3x + 4$  
D $y = 3x + 1$

23. USED CARS Gianna wants to buy a specific make and model of a used car. She researched prices from dealers and private sellers and made the graph shown. (Lesson 4-5)

a. Describe the relationship in the data.

b. Use the line of fit to predict the price of a car that is 7 years old.

c. Is it reasonable to use this line of fit to predict the price of a 10-year-old car? Explain.

24. GEOMETRY A quadrilateral has sides with equations $y = -2x$, $2x + y = 6$, $y = \frac{1}{2}x + 6$, and $x - 2y = 9$. Is the figure a rectangle? Explain your reasoning. (Lesson 4-4)

Write each equation in standard form. (Lesson 4-3)

25. $y - 2 = 3(x - 1)$  
26. $y - 5 = 6(x + 1)$  
27. $y + 2 = -2(x - 5)$  
28. $y + 3 = \frac{1}{2}(x + 4)$  
29. $y - 1 = \frac{2}{3}(x + 9)$  
30. $y + 3 = -\frac{1}{4}(x + 2)$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

31. $(3, 4), (10, 8)$  
32. $(-4, 7), (3, 5)$  
33. $(3, 7), (-2, 4)$  
34. $(-3, 2), (-3, 4)$

Skills Review

If $f(x) = x^2 - x + 1$, find each value.

35. $f(-1)$  
36. $f(5) - 3$  
37. $f(a)$  
38. $f(b^2)$

Graph each equation.

39. $y = x + 2$  
40. $x + 5y = 4$  
41. $2x - 3y = 6$  
42. $5x + 2y = 6$
Inverse Linear Functions

New Vocabulary
inverse relation
inverse function

1 Inverse Relations

An inverse relation is the set of ordered pairs obtained by exchanging the $x$-coordinates with the $y$-coordinates of each ordered pair in a relation. If $(5, 3)$ is an ordered pair of a relation, then $(3, 5)$ is an ordered pair of the inverse relation.

Key Concept Inverse Relations

Words
If one relation contains the element $(a, b)$, then the inverse relation will contain the element $(b, a)$.

Example

$A$ and $B$ are inverse relations.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-3, -16)$</td>
<td>$(-16, -3)$</td>
</tr>
<tr>
<td>$(-1, 4)$</td>
<td>$(4, -1)$</td>
</tr>
<tr>
<td>$(2, 14)$</td>
<td>$(14, 2)$</td>
</tr>
<tr>
<td>$(5, 32)$</td>
<td>$(32, 5)$</td>
</tr>
</tbody>
</table>

Notice that the domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

Example 1 Inverse Relations

Find the inverse of each relation.

a. $\{(4, -10), (7, -19), (-5, 17), (-3, 11)\}$

To find the inverse, exchange the coordinates of the ordered pairs.

$(4, -10) \rightarrow (-10, 4)$

$(7, -19) \rightarrow (-19, 7)$

$(5, 17) \rightarrow (17, -5)$

$(-3, 11) \rightarrow (11, -3)$

The inverse is $\{(-10, 4), (-19, 7), (17, -5), (11, -3)\}$.

b. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>$-1$</th>
<th>$5$</th>
<th>$9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-13$</td>
<td>$-8.5$</td>
<td>$0.5$</td>
<td>$6.5$</td>
</tr>
</tbody>
</table>

Write the coordinates as ordered pairs. Then exchange the coordinates of each pair.

$(-4, -13) \rightarrow (-13, -4)$

$(5, 0.5) \rightarrow (0.5, 5)$

$(-1, -8.5) \rightarrow (-8.5, -1)$

$(9, 6.5) \rightarrow (6.5, 9)$

The inverse is $\{(-13, -4), (-8.5, -1), (0.5, 5), (6.5, 9)\}$. 
Guided Practice

1A. \{(-6, 8), (-15, 11), (9, 3), (0, 6)\}

1B. \[
\begin{array}{c|cccc}
  x & -10 & -4 & -3 & 0 \\
  y & 5 & 11 & 12 & 15 \\
\end{array}
\]

The graphs of relations can be used to find and graph inverse relations.

Example 2 Graph Inverse Relations

Graph the inverse of the relation.

The graph of the relation passes through the points at \((-4, -3), (-2, -1), (0, 1), (2, 3), \) and \((3, 4)\). To find points through which the graph of the inverse passes, exchange the coordinates of the ordered pairs. The graph of the inverse passes through the points at \((-3, -4), (-1, -2), (1, 0), (3, 2), \) and \((4, 3)\). Graph these points and then draw the line that passes through them.

Guided Practice

Graph the inverse of each relation.

2A.

2B.

The graphs from Example 2 are graphed on the right with the line \(y = x\). Notice that the graph of an inverse is the graph of the original relation reflected in the line \(y = x\). For every point \((x, y)\) on the graph of the original relation, the graph of the inverse will include the point \((y, x)\).

Inverse Functions

A linear relation that is described by a function has an inverse function that can generate ordered pairs of the inverse relation. The inverse of the linear function \(f(x)\) can be written as \(f^{-1}(x)\) and is read \(f\) of \(x\) inverse or the inverse of \(f\) of \(x\).
**Key Concept** Finding Inverse Functions

To find the inverse function \( f^{-1}(x) \) of the linear function \( f(x) \), complete the following steps.

**Step 1** Replace \( f(x) \) with \( y \) in the equation for \( f(x) \).

**Step 2** Interchange \( y \) and \( x \) in the equation.

**Step 3** Solve the equation for \( y \).

**Step 4** Replace \( y \) with \( f^{-1}(x) \) in the new equation.

**Example 3** Find Inverse Linear Functions

Find the inverse of each function.

**a.** \( f(x) = 4x - 8 \)

**Step 1** \( f(x) = 4x - 8 \)  
Original equation  
\( y = 4x - 8 \)  
Replace \( f(x) \) with \( y \).

**Step 2** \( x = 4y - 8 \)  
Interchange \( y \) and \( x \).

**Step 3** \( x + 8 = 4y \)  
Add 8 to each side.  
\( \frac{x + 8}{4} = y \)  
Divide each side by 4.

**Step 4** \( \frac{x + 8}{4} = f^{-1}(x) \)  
Replace \( y \) with \( f^{-1}(x) \).

The inverse of \( f(x) = 4x - 8 \) is \( f^{-1}(x) = \frac{x + 8}{4} \) or \( f^{-1}(x) = \frac{1}{4}x + 2 \).

**CHECK** Graph both functions and the line \( y = x \) on the same coordinate plane. \( f^{-1}(x) \) appears to be the reflection of \( f(x) \) in the line \( y = x \).

**b.** \( f(x) = -\frac{1}{2}x + 11 \)

**Step 1** \( f(x) = -\frac{1}{2}x + 11 \)  
Original equation  
\( y = -\frac{1}{2}x + 11 \)  
Replace \( f(x) \) with \( y \).

**Step 2** \( x = -\frac{1}{2}y + 11 \)  
Interchange \( y \) and \( x \).

**Step 3** \( x - 11 = -\frac{1}{2}y \)  
Subtract 11 from each side.  
\( -2(x - 11) = y \)  
Multiply each side by \(-2\).  
\( -2x + 22 = y \)  
Distributive Property

**Step 4** \( -2x + 22 = f^{-1}(x) \)  
Replace \( y \) with \( f^{-1}(x) \).

The inverse of \( f(x) = -\frac{1}{2}x + 11 \) is \( f^{-1}(x) = -2x + 22 \).

**Guided Practice**

3A. \( f(x) = 4x - 12 \)  
3B. \( f(x) = \frac{1}{3}x + 7 \)
**Real-World Example 4** Use an Inverse Function

**TEMPERATURE** Refer to the beginning of the lesson. Randall wants to convert the temperatures from degrees Celsius to degrees Fahrenheit.

a. Find the inverse function $C^{-1}(x)$.

**Step 1**

\[ C(x) = \frac{5}{9} (x - 32) \quad \text{Original equation} \]

\[ y = \frac{5}{9} (x - 32) \quad \text{Replace } C(x) \text{ with } y. \]

**Step 2**

\[ x = \frac{5}{9} (y - 32) \quad \text{Interchange } y \text{ and } x. \]

**Step 3**

\[ \frac{9}{5}x = y - 32 \]

\[ \frac{9}{5}x + 32 = y \quad \text{Multiply each side by } \frac{9}{5}. \]

**Step 4**

\[ \frac{9}{5}x + 32 = C^{-1}(x) \quad \text{Replace } y \text{ with } C^{-1}(x). \]

The inverse function of $C(x)$ is $C^{-1}(x) = \frac{9}{5}x + 32$.

b. What do $x$ and $C^{-1}(x)$ represent in the context of the inverse function?

$x$ represents the temperature in degrees Celsius. $C^{-1}(x)$ represents the temperature in degrees Fahrenheit.

c. Find the average temperatures for July in degrees Fahrenheit.

The average minimum and maximum temperatures for July are $3^\circ C$ and $15^\circ C$, respectively. To find the average minimum temperature, find $C^{-1}(3)$.

\[ C^{-1}(x) = \frac{9}{5}x + 32 \quad \text{Original equation} \]

\[ C^{-1}(3) = \frac{9}{5}(3) + 32 \quad \text{Substitute 3 for } x. \]

\[ = 37.4 \quad \text{Simplify.} \]

To find the average maximum temperature, find $C^{-1}(15)$.

\[ C^{-1}(x) = \frac{9}{5}x + 32 \quad \text{Original equation} \]

\[ C^{-1}(15) = \frac{9}{5}(15) + 32 \quad \text{Substitute 15 for } x. \]

\[ = 59 \quad \text{Simplify.} \]

The average minimum and maximum temperatures for July are $37.4^\circ F$ and $59^\circ F$, respectively.

**Guided Practice**

4. **RENTAL CAR** Peggy rents a car for the day. The total cost $C(x)$ in dollars is given by $C(x) = 19.99 + 0.3x$, where $x$ is the number of miles she drives.

A. Find the inverse function $C^{-1}(x)$.

B. What do $x$ and $C^{-1}(x)$ represent in the context of the inverse function?

C. How many miles did Peggy drive if her total cost was $34.99?
Check Your Understanding

Example 1  
Find the inverse of each relation.
1. \{(4, -15), (-8, -18), (-2, -16.5), (3, -15.25)\}

2. 
\[
\begin{array}{c|c|c|c|c}
 x & -3 & 0 & 1 & 6 \\
 y & 11.8 & 3.7 & 1 & -12.5 \\
\end{array}
\]

Example 2  
Graph the inverse of each relation.
3. [Graph of a line]
4. [Graph of a line]

Example 3  
Find the inverse of each function.
5. \(f(x) = -2x + 7\)
6. \(f(x) = \frac{2}{3}x + 6\)

Example 4  
7. **REASONING**  
Dwayne and his brother purchase season tickets to the Cleveland Crusaders games. The ticket package requires a one-time purchase of a personal seat license costing $1200 for two seats. A ticket to each game costs $70. The cost \(C(x)\) in dollars for Dwayne for the first season is \(C(x) = 600 + 70x\), where \(x\) is the number of games Dwayne attends.
   a. Find the inverse function.
   b. What do \(x\) and \(C^{-1}(x)\) represent in the context of the inverse function?
   c. How many games did Dwayne attend if his total cost for the season was $950?

Practice and Problem Solving

Example 1  
Find the inverse of each relation.
8. \{(-5, 13), (6, 10.8), (3, 11.4), (-10, 14)\}
9. \{(-4, -49), (8, 35), (-1, -28), (4, 7)\}

10. 
\[
\begin{array}{c|c|c}
 x & y \\
-8 & -36.4 \\
-2 & -15.4 \\
1 & -4.9 \\
5 & 9.1 \\
11 & 30.1 \\
\end{array}
\]

11. 
\[
\begin{array}{c|c}
 x & y \\
-3 & 7.4 \\
-1 & 4 \\
1 & 0.6 \\
3 & -2.8 \\
5 & -6.2 \\
\end{array}
\]

Example 2  
Graph the inverse of each relation.
12. [Graph of a line]
13. [Graph of a line]
Example 3  Find the inverse of each function.

14. \( f(x) = 25 + 4x \)  \hspace{1cm} 15. \( f(x) = 17 - \frac{1}{3}x \)

16. \( f(x) = 4(x + 17) \)  \hspace{1cm} 17. \( f(x) = 12 - 6x \)

18. \( f(x) = \frac{2}{3}x + 10 \)  \hspace{1cm} 19. \( f(x) = -16 - \frac{4}{3}x \)

Example 4  

20. **DOWNLOADS**  An online music subscription service allows members to download songs for $0.99 each after paying a monthly service charge of $3.99. The total monthly cost \( C(x) \) of the service in dollars is \( C(x) = 3.99 + 0.99x \), where \( x \) is the number of songs downloaded.
   a. Find the inverse function.
   b. What do \( x \) and \( C^{-1}(x) \) represent in the context of the inverse function?
   c. How many songs were downloaded if a member’s monthly bill is $27.75?

21. **LANDSCAPING**  At the start of the mowing season, Chuck collects a one-time maintenance fee of $10 from his customers. He charges the Fosters $35 for each cut. The total amount collected from the Fosters in dollars for the season is \( C(x) = 10 + 35x \), where \( x \) is the number of times Chuck mows the Fosters’ lawn.
   a. Find the inverse function.
   b. What do \( x \) and \( C^{-1}(x) \) represent in the context of the inverse function?
   c. How many times did Chuck mow the Fosters’ lawn if he collected a total of $780 from them?

Write the inverse of each equation in \( f^{-1}(x) \) notation.

22. \( 3y - 12x = -72 \)  \hspace{1cm} 23. \( x + 5y = 15 \)

24. \( -42 + 6y = x \)  \hspace{1cm} 25. \( 3y + 24 = 2x \)

26. \( -7y + 2x = -28 \)  \hspace{1cm} 27. \( 3y - x = 3 \)

**CCSS** **TOOLS**  Match each function with the graph of its inverse.

A. \( f(x) \)  \hspace{1cm} B. \( f(x) \)

C. \( f(x) \)  \hspace{1cm} D. \( f(x) \)

28. \( f(x) = x + 4 \)  \hspace{1cm} 29. \( f(x) = 4x + 4 \)

30. \( f(x) = \frac{1}{4}x + 1 \)  \hspace{1cm} 31. \( f(x) = \frac{1}{4}x - 1 \)
Write an equation for the inverse function $f^{-1}(x)$ that satisfies the given conditions.

32. slope of $f(x)$ is 7; graph of $f^{-1}(x)$ contains the point (13, 1)

33. graph of $f(x)$ contains the points $(-3, 6)$ and $(6, 12)$

34. graph of $f(x)$ contains the point $(10, 16)$; graph of $f^{-1}(x)$ contains the point $(3, -16)$

35. slope of $f(x)$ is 4; $f^{-1}(5) = 2$

36. **CELL PHONES** Mary Ann pays a monthly fee for her cell phone package which includes 700 minutes. She gets billed an additional charge for every minute she uses the phone past the 700 minutes. During her first month, Mary Ann used 26 additional minutes and her bill was $37.79. During her second month, Mary Ann used 38 additional minutes and her bill was $41.39.

   a. Write a function that represents the total monthly cost $C(x)$ of Mary Ann’s cell phone package, where $x$ is the number of additional minutes used.

   b. Find the inverse function.

   c. What do $x$ and $C^{-1}(x)$ represent in the context of the inverse function?

   d. How many additional minutes did Mary Ann use if her bill for her third month was $48.89?

37. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the domain and range of inverse functions.

   a. **Algebraic** Write a function for the area $A(x)$ of the rectangle shown.

   b. **Graphical** Graph $A(x)$. Describe the domain and range of $A(x)$ in the context of the situation.

   c. **Algebraic** Write the inverse of $A(x)$. What do $x$ and $A^{-1}(x)$ represent in the context of the situation?

   d. **Graphical** Graph $A^{-1}(x)$. Describe the domain and range of $A^{-1}(x)$ in the context of the situation.

   e. **Logical** Determine the relationship between the domains and ranges of $A(x)$ and $A^{-1}(x)$.

**H.O.T. Problems** Use Higher-Order Thinking Skills

38. **CHALLENGE** If $f(x) = 5x + a$ and $f^{-1}(10) = -1$, find $a$.

39. **CHALLENGE** If $f(x) = \frac{1}{a}x + 7$ and $f^{-1}(x) = 2x - b$, find $a$ and $b$.

**ARGUMENTS** Determine whether the following statements are sometimes, always, or never true. Explain your reasoning.

40. If $f(x)$ and $g(x)$ are inverse functions, then $f(a) = b$ and $g(b) = a$.

41. If $f(a) = b$ and $g(b) = a$, then $f(x)$ and $g(x)$ are inverse functions.

42. **OPEN ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses by graphing the functions and the line $y = x$ on the same coordinate plane.

43. **WRITING IN MATH** Explain why it may be helpful to find the inverse of a function.
44. Which equation represents a line that is perpendicular to the graph and passes through the point at (2, 0)?
   A \( y = 3x - 6 \)
   B \( y = -3x + 6 \)
   C \( y = -\frac{1}{3}x + \frac{2}{3} \)
   D \( y = \frac{1}{3}x - \frac{2}{3} \)

45. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time \( t \) it would take the giant tortoise to travel 0.8 mile?
   F \( t = \frac{0.8}{0.17} \)
   H \( t = \frac{0.17}{0.8} \)
   G \( t = (0.17)(0.8) \)
   J \( 0.8 = \frac{0.17}{t} \)

46. GEOMETRY If \( \triangle JKL \) is similar to \( \triangle JNM \) what is the value of \( a \)?
   A 62.5
   B 105
   C 125
   D 155.5

47. GRIDDED RESPONSE What is the difference in the value of \( 2.1(x + 3.2) \), when \( x = 5 \) and when \( x = 3 \)?

Spiral Review

Write an equation of the regression line for the data in each table. (Lesson 4-6)

48. \[
\begin{array}{c|c|c|c|c|c}
 x & 1 & 3 & 5 & 7 & 9 \\
 y & 3 & 8 & 15 & 18 & 21 \\
\end{array}
\]

49. \[
\begin{array}{c|c|c|c|c|c}
 x & 3 & 5 & 7 & 9 & 11 \\
 y & 7.2 & 23.5 & 41.2 & 56.4 & 73.1 \\
\end{array}
\]

50. \[
\begin{array}{c|c|c|c|c|c}
 x & 1 & 2 & 3 & 4 & 5 \\
 y & 21 & 33 & 39 & 54 & 64 \\
\end{array}
\]

51. \[
\begin{array}{c|c|c|c|c|c}
 x & 2 & 4 & 6 & 8 & 10 \\
 y & 1.4 & 2.4 & 2.9 & 3.3 & 4.2 \\
\end{array}
\]

52. TESTS Determine whether the graph at the right shows a positive, negative, or no correlation. If there is a correlation, describe its meaning. (Lesson 4-5)

Suppose \( y \) varies directly as \( x \). (Lesson 3-4)

53. If \( y = 2.5 \) when \( x = 0.5 \), find \( y \) when \( x = 20 \).
54. If \( y = -6.6 \) when \( x = 9.9 \), find \( y \) when \( x = 6.6 \).
55. If \( y = 2.6 \) when \( x = 0.25 \), find \( y \) when \( x = 1.125 \).
56. If \( y = 6 \) when \( x = 0.6 \), find \( x \) when \( y = 12 \).

Skills Review

Solve each equation.

57. \( 104 = k - 67 \)
58. \( -4 + x = -7 \)
59. \( \frac{m}{7} = -11 \)
60. \( \frac{2}{3}p = 14 \)
61. \( -82 = 18 - n \)
62. \( \frac{9}{t} = -27 \)
You can use patty paper to draw the graph of an inverse relation by reflecting the original graph in the line \( y = x \).

**Activity**  
**Draw an Inverse**

Consider the graphs shown.

**Step 1** Trace the graphs onto a square of patty paper, waxed paper, or tracing paper.

**Step 2** Flip the patty paper over and lay it on the original graph so that the traced \( y = x \) is on the original \( y = x \).

Notice that the result is the reflection of the graph in the line \( y = x \) or the inverse of the graph.

**Analyze The Results**

1. Is the graph of the original relation a function? Explain.

2. Is the graph of the inverse relation a function? Explain.

3. What are the domain and range of the original relation? of the inverse relation?

4. If the domain of the original relation is restricted to \( D = \{ x \mid x \geq 0 \} \), is the inverse relation a function? Explain.

5. If the graph of a relation is a function, what can you conclude about the graph of its inverse?

6. **CHALLENGE** The vertical line test can be used to determine whether a relation is a function. Write a rule that can be used to determine whether a function has an inverse that is also a function.
Study Guide

**Key Concepts**

**Slope-Intercept Form** (Lessons 4-1 and 4-2)
- The slope-intercept form of a linear equation is \( y = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept.
- If you are given two points through which a line passes, use them to find the slope first.

**Point-Slope Form** (Lesson 4-3)
- The linear equation \( y - y_1 = m(x - x_1) \) is written in point-slope form, where \((x_1, y_1)\) is a given point on a nonvertical line and \( m \) is the slope of the line.

**Parallel and Perpendicular Lines** (Lesson 4-4)
- Nonvertical parallel lines have the same slope.
- Lines that intersect at right angles are called perpendicular lines. The slopes of perpendicular lines are opposite reciprocals.

**Scatter Plots and Lines of Fit** (Lesson 4-5)
- Data with two variables are called bivariate data.
- A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

**Regression and Median-Fit Lines** (Lesson 4-6)
- A graphing calculator can be used to find regression lines and median-fit lines.

**Inverse Linear Functions** (Lesson 4-7)
- An inverse relation is the set of ordered pairs obtained by exchanging the \( x \)-coordinates with the \( y \)-coordinates of each ordered pair of a relation.
- A linear function \( f(x) \) has an inverse function that can be written as \( f^{-1}(x) \) and is read \( f \) of \( x \) inverse or the inverse of \( f \) of \( x \).

**Key Vocabulary**

- best-fit line (p. 255)
- linear interpolation (p. 249)
- bivariate data (p. 247)
- linear regression (p. 255)
- constant function (p. 217)
- line of fit (p. 248)
- constraint (p. 228)
- median-fit line (p. 258)
- correlation coefficient (p. 255)
- parallel lines (p. 239)
- identity function (p. 224)
- perpendicular lines (p. 240)
- inverse function (p. 264)
- point-slope form (p. 233)
- inverse relation (p. 263)
- scatter plot (p. 247)
- linear extrapolation (p. 228)
- slope-intercept form (p. 216)

**Vocabulary Check**

State whether each sentence is true or false. If false, replace the underlined term to make a true sentence.

1. The \( y \)-intercept is the \( y \)-coordinate of the point where the graph crosses the \( y \)-axis.
2. The process of using a linear equation to make predictions about values that are beyond the range of the data is called linear regression.
3. An inverse relation is the set of ordered pairs obtained by exchanging the \( x \)-coordinates with the \( y \)-coordinates of each ordered pair of a relation.
4. The correlation coefficient describes whether the correlation between the variables is positive or negative and how closely the regression equation is modeling the data.
5. Lines in the same plane that do not intersect are called parallel lines.
6. Lines that intersect at acute angles are called perpendicular lines.
7. A(n) constant function can generate ordered pairs for an inverse relation.
8. The range of a relation is the range of its inverse function.
9. An equation of the form \( y = mx + b \) is in point-slope form.
Lesson-by-Lesson Review

4-1 Graphing Equations in Slope-Intercept Form

Write an equation of a line in slope-intercept form with the given slope and y-intercept. Then graph the equation.

10. slope: 3, y-intercept: 5
11. slope: −2, y-intercept: −9
12. slope: \( \frac{2}{3} \), y-intercept: 3
13. slope: \( -\frac{5}{8} \), y-intercept: −2

Graph each equation.

14. \( y = 4x - 2 \)
15. \( y = -3x + 5 \)
16. \( y = \frac{1}{2}x + 1 \)
17. \( 3x + 4y = 8 \)

18. **Ski Rental** Write an equation in slope-intercept form for the total cost of skiing for \( h \) hours with one lift ticket.

**Example 1**

Write an equation of a line in slope-intercept form with slope \( -5 \) and y-intercept \( -3 \). Then graph the equation.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
y = -5x + (-3) \quad m = -5 \text{ and } b = -3
\]

To graph the equation, plot the y-intercept (0, −3).

Then move up 5 units and left 1 unit. Plot the point.

Draw a line through the two points.

4-2 Writing Equations in Slope-Intercept Form

Write an equation of the line that passes through the given point and has the given slope.

19. (1, 2), slope 3
20. (2, −6), slope −4
21. (−3, −1), slope \( \frac{2}{3} \)
22. (5, −2), slope \( -\frac{1}{3} \)

Write an equation of the line that passes through the given points.

23. (2, −1), (5, 2)
24. (−4, 3), (1, 13)
25. (3, 5), (5, 6)
26. (2, 4), (7, 2)

27. **Camp** In 2005, a camp had 450 campers. Five years later, the number of campers rose to 750. Write a linear equation that represents the number of campers that attend camp.

**Example 2**

Write an equation of the line that passes through (3, 2) with a slope of 5.

**Step 1** Find the y-intercept.

\[
y = mx + b \quad \text{Slope-intercept form}
\]

\[
2 = 5(3) + b \quad m = 5, y = 2, \text{ and } x = 3
\]

\[
2 = 15 + b \quad \text{Simplify.}
\]

\[
-13 = b \quad \text{Subtract 15 from each side.}
\]

**Step 2** Write the equation in slope-intercept form.

\[
y = 5x - 13 \quad m = 5 \text{ and } b = -13
\]
4-3 Writing Equations in Point-Slope Form

Write an equation in point-slope form for the line that passes through the given point with the slope provided.

28. (6, 3), slope 5
29. (−2, 1), slope −3
30. (−4, 2), slope 0

Write each equation in standard form.

31. \( y - 3 = 5(x - 2) \)
32. \( y - 7 = -3(x + 1) \)
33. \( y + 4 = \frac{1}{2}(x - 3) \)
34. \( y - 9 = -\frac{4}{5}(x + 2) \)

Write each equation in slope-intercept form.

35. \( y - 2 = 3(x - 5) \)
36. \( y - 12 = -2(x - 3) \)
37. \( y + 3 = 5(x + 1) \)
38. \( y - 4 = \frac{1}{2}(x + 2) \)

Example 3

Write an equation in point-slope form for the line that passes through (3, 4) with a slope of −2.

\[ y - y_1 = m(x - x_1) \]
\[ y - 4 = -2(x - 3) \]

Replace \( m \) with −2 and \( (x_1, y_1) \) with (3, 4).

Example 4

Write \( y + 6 = -4(x - 3) \) in standard form.

\[ y + 6 = -4(x - 3) \] Original equation
\[ y + 6 = -4x + 12 \] Distributive Property
\[ 4x + y + 6 = 12 \] Add 4x to each side.
\[ 4x + y = 6 \] Subtract 6 from each side.

4-4 Parallel and Perpendicular Lines

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

39. (2, 5), \( y = x - 3 \)
40. (0, 3), \( y = 3x + 5 \)
41. (−4, 1), \( y = -2x - 6 \)
42. (−5, −2), \( y = -\frac{1}{2}x + 4 \)

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the given equation.

43. (2, 4), \( y = 3x + 1 \)
44. (1, 3), \( y = -2x - 4 \)
45. (−5, 2), \( y = \frac{1}{3}x + 4 \)
46. (3, 0), \( y = -\frac{1}{2}x \)

Example 5

Write an equation in slope-intercept form for the line that passes through (−2, 4) and is parallel to the graph of \( y = 6x - 3 \).

The slope of the line with equation \( y = 6x - 3 \) is 6. The line parallel to \( y = 6x - 3 \) has the same slope, 6.

\[ y - y_1 = m(x - x_1) \] Point-slope form
\[ y - 4 = 6[x - (-2)] \] Substitute.
\[ y - 4 = 6(x + 2) \] Simplify.
\[ y - 4 = 6x + 12 \] Distributive Property
\[ y = 6x + 16 \] Add 4 to each side.
47. Determine whether the graph shows a positive, negative, or no correlation. If there is a positive or negative correlation, describe its meaning.

48. ATTENDANCE  A scatter plot of data compares the number of years since a business has opened and its annual number of sales. It contains the ordered pairs (2, 650) and (5, 1280). Write an equation in slope-intercept form for the line of fit for this situation.

Example 6
The scatter plot displays the number of texts and the number of calls made daily. Write an equation for the line of fit.

First, find the slope using (2, 9) and (17, 4).

\[ m = \frac{4 - 9}{17 - 2} = \frac{-5}{15} \text{ or } -\frac{1}{3} \]

Substitute and simplify.

Then find the y-intercept.

\[ 9 = -\frac{1}{3}(2) + b \]

Substitute.

\[ 9\frac{2}{3} = b \]

Add \( \frac{2}{3} \) to each side.

Write the equation.

\[ y = -\frac{1}{3}x + 9\frac{2}{3} \]

49. SALE  The table shows the number of purchases made at an outerwear store during a sale. Write an equation of the regression line. Then estimate the daily purchases on day 10 of the sale.

<table>
<thead>
<tr>
<th>Days Since Sale Began</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Purchases</td>
<td>15</td>
<td>21</td>
<td>32</td>
<td>30</td>
<td>40</td>
<td>38</td>
<td>51</td>
</tr>
</tbody>
</table>

50. MOVIES  The table shows ticket sales at a certain theater during the first week after a movie opened. Write an equation of the regression line. Then estimate the daily ticket sales on the 15th day.

<table>
<thead>
<tr>
<th>Days Since Movie Opened</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Ticket Sales</td>
<td>85</td>
<td>92</td>
<td>89</td>
<td>78</td>
<td>65</td>
<td>68</td>
<td>55</td>
</tr>
</tbody>
</table>

Example 7
ATTENDANCE  The table shows the annual attendance at an amusement park. Write an equation of the regression line for the data.

<table>
<thead>
<tr>
<th>Years Since 2004</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance (thousands)</td>
<td>75</td>
<td>80</td>
<td>72</td>
<td>68</td>
<td>65</td>
<td>60</td>
<td>53</td>
</tr>
</tbody>
</table>

Step 1 Enter the data by pressing STAT and selecting the Edit option.

Step 2 Perform the regression by pressing STAT and selecting the CALC option. Scroll down to LinReg (aX + b) and press ENTER.

Step 3 Write the equation of the regression line by rounding the a- and b-values on the screen.

\[ y = -4.04x + 79.68 \]
### 4-7 Inverse Linear Functions

#### Find the inverse of each relation.

51. \( \{(7, 3.5), (6.2, 8), (-4, 2.7), (-12, 1.4)\} \)
52. \( \{(1, 9), (13, 26), (-3, 4), (-11, -2)\} \)
53. \( \begin{array}{c|c}
-4 & 2.7 \\
-1 & 3.8 \\
0 & 4.1 \\
3 & 7.2 \\
\end{array} \)
54. \( \begin{array}{c|c}
-12 & 4 \\
-8 & 0 \\
-4 & -4 \\
0 & -8 \\
\end{array} \)

#### Find the inverse of each function.

55. \( f(x) = \frac{5}{11} x + 10 \)
56. \( f(x) = 3x + 8 \)
57. \( f(x) = -4x - 12 \)
58. \( f(x) = \frac{1}{4} x - 7 \)
59. \( f(x) = -\frac{2}{3} x + \frac{1}{4} \)
60. \( f(x) = -3x + 3 \)

---

#### Example 8

Find the inverse of the relation.

\( \{(5, -3), (11, 2), (-6, 12), (4, -2)\} \)

To find the inverse, exchange the coordinates of the ordered pairs.

\[
(5, -3) \rightarrow (-3, 5) \\
(11, 2) \rightarrow (2, 11) \\
(-6, 12) \rightarrow (12, -6) \\
(4, -2) \rightarrow (-2, 4)
\]

The inverse is \( \{(-3, 5), (2, 11), (12, -6), (-2, 4)\} \).

#### Example 9

Find the inverse of \( f(x) = \frac{1}{4} x + 9 \).

\[
f(x) = \frac{1}{4} x + 9 \\
y = \frac{1}{4} x + 9 \\
x = \frac{1}{4} y + 9 \\
x - 9 = \frac{1}{4} y \\
4(x - 9) = y \\
4x - 36 = y \\
4x - 36 = f^{-1}(x)
\]

Replace \( y \) with \( f^{-1}(x) \).
1. Graph \( y = 2x - 3 \).

2. **MULTIPLE CHOICE** A popular pizza parlor charges $12 for a large cheese pizza plus $1.50 for each additional topping. Write an equation in slope-intercept form for the total cost \( C \) of a pizza with \( t \) toppings.
   - A \( C = 12t + 1.50 \)
   - B \( C = 13.50t \)
   - C \( C = 12 + 1.50t \)
   - D \( C = 1.50t - 12 \)

Write an equation of a line in slope-intercept form that passes through the given point and has the given slope.

3. \((-4, 2); \) slope \(-3\)
4. \((3, -5); \) slope \(\frac{2}{3}\)

Write an equation of the line in slope-intercept form that passes through the given points.

5. \((1, 4), (3, 10)\)
6. \((2, 5), (-2, 8)\)
7. \((0, 4), (-3, 0)\)
8. \((7, -1), (9, -4)\)

9. **PAINTING** The data in the table show the size of a room in square feet and the time it takes to paint the room in minutes.

<table>
<thead>
<tr>
<th>Room Size</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>400</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Painting Time</td>
<td>160</td>
<td>220</td>
<td>270</td>
<td>500</td>
<td>680</td>
</tr>
</tbody>
</table>

a. Use the points \((100, 160)\) and \((500, 680)\) to write an equation in slope-intercept form.
b. Predict the amount of time required to paint a room measuring 750 square feet.

10. **SALARY** The table shows the relationship between years of experience and teacher salary.

<table>
<thead>
<tr>
<th>Years Experience</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary (thousands of dollars)</td>
<td>28</td>
<td>31</td>
<td>42</td>
<td>49</td>
<td>64</td>
</tr>
</tbody>
</table>

a. Write an equation for the best-fit line.
b. Find the correlation coefficient and explain what it tells us about the relationship between experience and salary.

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

11. \((2, -3); y = 4x - 9\)
12. \((-5, 1); y = -3x + 2\)

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

13. \((1, 4); y = -2x + 5\)
14. \((-3, 6); y = \frac{1}{4}x + 2\)

15. **MULTIPLE CHOICE** The graph shows the relationship between outside temperature and daily ice cream cone sales. What type of correlation is shown?

F positive correlation
G negative correlation
H no correlation
J not enough information

16. **ADOPTION** The table shows the number of children from Ethiopia adopted by U.S. citizens.

<table>
<thead>
<tr>
<th>Years Since 2000</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Children</td>
<td>442</td>
<td>731</td>
<td>1254</td>
<td>1724</td>
<td>2277</td>
</tr>
</tbody>
</table>

a. Write the slope-intercept form of the equation for the line of fit.
b. Predict the number of children from Ethiopia who will be adopted in 2025.

Find the inverse of each function.

17. \(f(x) = -5x - 30\)
18. \(f(x) = 4x + 10\)
19. \(f(x) = \frac{1}{6}x - 2\)
20. \(f(x) = \frac{3}{4}x + 12\)
Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.

Strategies for Solving Short Answer Questions

Step 1

Short answer questions are typically graded using a rubric, or a scoring guide. The following is an example of a short answer question scoring rubric.

<table>
<thead>
<tr>
<th>Scoring Rubric</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Criteria</td>
<td></td>
</tr>
<tr>
<td>Full Credit: The answer is correct and a full explanation is provided that shows each step.</td>
<td>2</td>
</tr>
<tr>
<td>Partial Credit:</td>
<td></td>
</tr>
<tr>
<td>• The answer is correct, but the explanation is incomplete.</td>
<td>1</td>
</tr>
<tr>
<td>• The answer is incorrect, but the explanation is correct.</td>
<td></td>
</tr>
<tr>
<td>No Credit: Either an answer is not provided or the answer does not make sense.</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 2

In solving short answer questions, remember to...

- explain your reasoning or state your approach to solving the problem.
- show all of your work or steps.
- check your answer if time permits.

Standardized Test Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

The table shows production costs for building different numbers of skateboards. Determine the missing value, $x$, that will result in a linear model.

<table>
<thead>
<tr>
<th>Skateboards Built</th>
<th>Production Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>$325</td>
</tr>
<tr>
<td>28</td>
<td>$500</td>
</tr>
<tr>
<td>$x$</td>
<td>$375</td>
</tr>
<tr>
<td>22</td>
<td>$425</td>
</tr>
</tbody>
</table>
Read the problem carefully. You are given several data points and asked to find the missing value that results in a linear model.

Example of a 2-point response:

Set up a coordinate grid and plot the three given points: (14, 325), (28, 500), (22, 425).

Then draw a straight line through them and find the \(x\)-value that produces a \(y\)-value of 375.

So, building 18 skateboards would result in production costs of $375. These data form a linear model.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

**Exercises**

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

1. Given points \(M(-1, 7), N(3, -5), O(6, 1),\) and \(P(-3, -2),\) determine two segments that are perpendicular to each other.

2. Write the equation of a line that is parallel to \(4x + 2y = 8\) and has a \(y\)-intercept of 5.

3. Three vertices of a quadrilateral are shown on the coordinate grid. Determine a fourth vertex that would result in a trapezoid.
Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the rate of change represented in the graph?

![Graph showing a line with points (0, 4) and (5, -2).]

A $\frac{-2}{5}$  
B $\frac{-5}{6}$  
C $\frac{6}{5}$  
D $\frac{5}{2}$

2. The table below shows the cost for renting a bicycle at a bike shop located in Venice Beach. What is a function that can represent this sequence?

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>22</td>
</tr>
</tbody>
</table>

F $f(n) = 4n + 10$  
G $f(n) = 4n + 6$  
H $f(n) = 10n + 4$  
J $f(n) = 10n - 6$

3. Jaime bought a car in 2005 for $28,500. By 2008, the car was worth $23,700. Based on a linear model, what will the value of the car be in 2012?

A $17,300$  
B $17,550$  
C $18,100$  
D $18,475$

4. If the graph of a line has a positive slope and a negative y-intercept, what happens to the x-intercept if the slope and the y-intercept are doubled?

F The x-intercept becomes four times as great.  
G The x-intercept becomes twice as great.  
H The x-intercept becomes one-fourth as great.  
J The x-intercept remains the same.

5. Which absolute value equation has the graph below as its solution?

![Graph showing a V-shaped graph from x = -3 to x = 3.]

A $|x - 3| = 11$  
B $|x - 4| = 12$  
C $|x - 11| = 3$  
D $|x - 12| = 4$

6. The table below shows the relationship between certain temperatures in degrees Fahrenheit and degrees Celsius. Which of the following linear equations correctly models this relationship?

<table>
<thead>
<tr>
<th>Celsius (C)</th>
<th>Fahrenheit (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>50°</td>
</tr>
<tr>
<td>15°</td>
<td>59°</td>
</tr>
<tr>
<td>20°</td>
<td>68°</td>
</tr>
<tr>
<td>25°</td>
<td>77°</td>
</tr>
<tr>
<td>30°</td>
<td>86°</td>
</tr>
</tbody>
</table>

F $F = \frac{8}{5}C + 35$  
G $F = \frac{4}{5}C + 42$  
H $F = \frac{9}{5}C + 32$  
J $F = \frac{12}{5}C + 26$

Test-Taking Tip

Question 3 Find the average annual depreciation between 2005 and 2008. Then extend the pattern to find the car’s value in 2012.
7. What is the equation of the line graphed below?

Express your answer in point slope form using the point (−8, 3).

8. **GRIDDED RESPONSE** The linear equation below is a best fit model for the peak depth of the Mad River when \( x \) inches of rain fall. What would you expect the peak depth of the river to be after a storm that produces \( 1\frac{3}{4} \) inches of rain? Round your answer to the nearest tenth of a foot if necessary.

\[ y = 2.5x + 14.8 \]

9. Jacob formed an advertising company in 1992. Initially, the company only had 14 employees. In 2008, the company had grown to a total of 63 employees. Find the percent of change in the number of employees working at Jacob’s company. Round to the nearest tenth of a percent if necessary.

10. The table shows the total amount of rain during a storm.

<table>
<thead>
<tr>
<th>Hours</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches</td>
<td>0.45</td>
<td>0.9</td>
<td>1.35</td>
<td>1.8</td>
</tr>
</tbody>
</table>

a. Write an equation to fit the data in the table.

b. Describe the relationship between the hour and the amount of rain received.

11. An electrician charges a $25 consultation fee plus $35 per hour for labor.

a. Copy and complete the following table showing the charges for jobs that take 1, 2, 3, 4, or 5 hours.

<table>
<thead>
<tr>
<th>Hours, ( h )</th>
<th>Total Cost, ( C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

b. Write an equation in slope-intercept form for the total cost of a job that takes \( h \) hours.

c. If the electrician bills in quarter hours, how much would it cost for a job that takes 3 hours 15 minutes to complete?

12. Explain how you can determine whether two lines are parallel or perpendicular.